

LPV MODELS: IDENTIFICATION FOR GAIN SCHEDULING CONTROL

B. Bamieh*, L. Giarré†

* Mechanical and Environmental Engineering
University of California, Santa Barbara, CA 93106, USA.
bamieh@engineering.ucsb.edu (805)-893-4490

† Dipartimento di Ingegneria Automatica e Informatica
Università di Palermo, Palermo, Italy.
giarre@ias.unipa.it

Keywords: Gain scheduling control, identification for nonlinear systems, LPV models.

Abstract

In this paper the use of discrete-time Linear Parameter Varying (LPV) models for the gain scheduling control and identification methods for non-linear or time-varying system is considered. We report an overview on the existing literature on LPV systems for gain scheduling control and identification. Moreover, assuming that inputs, outputs and the scheduling parameters are measured, and a form of the functional dependence of the coefficients on the parameters is known, we show how the identification problem can be reduced to a linear regression so that a Least Mean Square and Recursive Least Square identification algorithm can be reformulated. Our methodology is applied for the identification of the LPV model of the stall and surge control for compressors of jet engines.

1 Introduction

In this paper we consider the problem of identifying discrete-time Linear Parameter Varying (LPV) models of non-linear or time-varying systems for gain scheduling control. We report an overview on LPV systems for gain scheduling control and identification. We show how the identification problem can be reduced to a linear regression and how the identification procedure can be applied to a stall and surge control problem for jet engine.

1.1 An overview on LPV systems for gain scheduling

In many practical situations, a control system is subject to transitions among different conditions associated to variations of some fundamental parameters:

- physical modifications of part of the plant
- modeling problem: when the linearization is applied to a system which may work in different operating points

One approach to deal with is the *gain scheduling*, when the compensator parameters are chosen on-line as function of the

plant parameters. A mayor drawback of linearization is that control design based on the linearized dynamics need not exhibit good performance or even stabilizing when operating away from the equilibrium. One possibility is to linearize along a trajectory which is not restricted to a local operating region. However, this trajectory must be known in advance in order to perform the control design, and such advance knowledge is often not available. *Gain Scheduling* is then a two-step procedure where: 1) one designs local linear controllers based on linearization of the nonlinear system at several different equilibria (operating conditions) 2) a global nonlinear controller for the nonlinear system is obtained by *interpolating* or *scheduling* among the local operations point design.

A useful paradigm to study gain scheduling is the one of LPV systems. The terminology Linear Parameter time-Varying (LPV) systems has been introduced in (Shamma and Xiong, 1999).

A discrete LPV systems is represented in state space as

$$\begin{aligned}x(t+1) &= A(p(t))x(t) + B(p(t))u(t) \\ y(t) &= C(p(t))x(t) + D(p(t))u(t)\end{aligned}$$

The exogenous parameter $p(t)$ is assumed *a priori* unknown. However, it can be measured or estimated upon operation of the system. The question is then why distinguish LPV from LTV systems? In LTV systems the time variations are known beforehand (as in periodic systems). In LPV systems, a-priori on the scheduling parameter are bounds on its magnitude or rate of change. Rather than model the dynamical evolution of a particular variable, one can treat it as an exogenous independent parameter, see (Shamma and Xiong, 1999), (Shamma, 1996), and the recent survey (Rugh and Shamma, 2000), and reference therein.

Suppose that the a priori is the range of variation on the exogenous parameter p . A possible control design is to consider the variation of p as uncertainty and to design a single robust controller for the family of systems, this is very conservative. It is convenient to measure the parameter value during operation to reduce conservatism: design robust controllers around each operating point and switch between controllers according to some gain-scheduling policy. This is a good compromise between performance and robustness and the stability question in the switching zone has been solved in (Shamma and Xiong, 1999).

An other approach of gain scheduling is the design of parameter dependent H_∞ controllers and the introduction of a LFT representation of the controllers dependent on the parameter $p(t)$ has been solved in (Packard, 1994).

In (Blanchini, 1999), a comparison between gain scheduling and the robust state feedback stabilization problem has been studied. Recently, for continuous systems, he showed that the two problems are the same, and the knowledge of the parameter $p(t)$ is not an advantage. However, he proved that the knowledge of the parameter allows for linear compensator, and that this equivalence is not true for discrete time systems.

Although the knowledge of the plant parameter is an advantage for the compensator, an interesting exception has been investigated in the literature which concerns the state feedback case. Indeed for the class of control-affine nonlinear continuous-time systems in which the term associated with the control is certain or the so called convex processes (Blanchini, 1999), the knowledge of the parameter is not an advantage for the compensator as far as it concerns its stabilization capability. As a particular case, gain scheduling design for state feedback LPV systems can be handled without restriction as a robust design, with the remarkable advantage that no parameter measurement are needed for stabilizable systems.

In the discrete-time case this property does not hold. Trivial examples exist of LPV discrete-time systems that can be stabilized via gain-scheduling controller but not by means of state feedback controllers which ignore the parameter. This facts motivated (Blanchini, 1994) to derive a procedure to design a gain scheduling control in which the exploitation of the parameter measurement is allowed.

Then, discrete time LPV systems are very important for the gain scheduling design, and the modeling, validation or identification oriented to the control plays a crucial role in the overall design.

1.2 Identification of LPV systems

We are considering techniques for the identification of nonlinear systems linearized along a varying trajectory. Let us assume that the system generating the data is described by the (possibly nonlinear) relationship

$$y = f(u, p, \eta) \quad (1)$$

where y , u and p are the measured experimental data, and η represents the unknown signals entering the system. In particular, y is the output, u is the input and p is the gain-scheduling parameter, a variable parameter that can be thought as determining the “set-point” of the system. In many process and aerospace control systems, it is a realistic assumption that the set point variable is measurable. As a *model* we consider a special class of discrete-time linear parameter-varying (LPV) models. The problem of identifying a linear parameter-varying systems has been studied in literature.

We refer the reader to (Campi, 1994), where a survey of the

literature has been presented for LTI models, and the exponentially weighted Least Squares algorithm and its stochastic properties have been studied. The model used is of the following form:

$$y_k = \psi_k^T \theta(k) + \eta_k \quad (2)$$

where η_k is white noise. Here, static nonlinearity are also considered (Hammerstein model), but the possibility of *dynamic* parameter-varying or in general LPV systems are not taken into account.

The recent literature on LPV models identification started with (Nemani *et al.*, 1995). This is the first attempt to solve the problem of identifying LPV systems, considering state measurements and one varying parameter. The problem was shown to be equivalent to a linear regression, and certain conservative conditions for persistency of excitation were given.

More closely related to our approach is the work of (Lee and Poolla, 1999), which addresses the problem of identifying a discrete LPV model

$$y_k = P(p, \theta)u_k + v_k$$

where p is a measured time-varying parameter, and it reduces the problem to one of nonlinear programming.

In (Previdi and Lovera, 1999), LPV models are identified with an hybrid linear/nonlinear procedure. The nonlinear part is identified through neural network and the linear parametric part through Least Square (LS) algorithm.

Finally, the approach in (Mazzaro *et al.*, 1999) is to consider a robust identification via worst-case identification.

The considered technique can also be improved with an iterative procedure that implement validation of the identified controlled model. Recently (in)validation scheme for LPV models has been presented in (Sznaier *et al.*, 2000).

In order to identify an input-output LPV model via linear regression we will use Least Mean Square (LMS) and Recursive Least Square (RLS) algorithms as developed in (Bamieh and Giarré, 1999), (Bamieh and Giarrè, 2000). This technique has the following advantage: in order to identify the multiple LTI models linearized along the trajectory as usually done in gain scheduling oriented identification scheme, the experiment here can be carried on in one shot, without requiring multiple experiment to collect data and without requiring that the set-point vary slowly.

Preliminaries

We will need the notions of inner and tensor products of matrices. Given any two matrices A, B of equal dimensions, their inner product is defined as

$$\langle A, B \rangle := \text{trace}(A^*B) = \text{trace}(BA^*),$$

where A^* means complex conjugate transpose of the matrix A . For given integers (n, m) , this inner product makes $\mathfrak{R}^{n \times m}$ (the space of $n \times m$) matrices into an inner product space.

Using the above inner product, one can define the tensor product of two matrices $U \in \mathbb{R}^{n \times m}$ and $V \in \mathbb{R}^{p \times q}$ as the linear operator $U \otimes V : \mathbb{R}^{p \times q} \rightarrow \mathbb{R}^{n \times m}$ defined by

$$(U \otimes V)(X) := U \langle V, X \rangle.$$

This definition generalizes the notion of outer products of vectors to matrices.

2 Problem formulation

Our special class of discrete-time LPV models is parameterized as follows

$$A(\delta, p) y(k) = B(\delta, p) u(k), \quad (3)$$

where y is the output and u is the input, δ is the delay operator i.e. $(\delta y)_k := y_{(k-1)}$, and

$$\begin{aligned} B(\delta, p) &:= b_0(p) + b_1(p)\delta + \dots + b_{n_b}(p)\delta^{n_b} \\ A(\delta, p) &:= 1 + a_1(p)\delta + \dots + a_{n_a}(p)\delta^{n_a} \end{aligned} \quad (4)$$

$n = n_a + n_b + 1$ is the number of parametric functions to be identified. Moreover, we assume that the varying parameter p is a function of discrete time ($p = p(k)$).

We assume that the parameters $\{a_i\}, \{b_j\}$ in (4) are in general known function of the parameter p of the form:

$$a_i(p) := \sum_{l=0}^{N-1} a_{il} f_l(p) \quad (5)$$

$$b_i(p) := \sum_{l=0}^{N-1} b_{il} f_l(p) \quad (6)$$

where the constants a_{il} are defined for $i = 1, \dots, n_a$ and b_{il} for $i = 0, \dots, n_b$.

Thus any particular model in our class is completely characterized by the real numbers $\{a_{ik}\}$ and $\{b_{il}\}$.

For this general framework, many choices are possible for the functions $f_l(p)$. In particular if we consider a polynomial dependence, then the functions $f_l(p)$ are simply powers of p

$$f_l(p) = p^{l-1} \quad l = 1, \dots, N$$

In this case the parameters are given by:

$$\begin{aligned} a_i(p) &= a_i^1 + a_i^2 p + \dots + a_i^N p^{N-1} \\ b_j(p) &= b_j^1 + b_j^2 p + \dots + b_j^N p^{N-1}. \end{aligned} \quad (7)$$

We now observe that the input-output model in (3), together with (6) and (4) can be put into a nice linear regression form as follows. Let us define an $n \times N$ matrix Θ which contains all

the coefficients to be identified:

$$\Theta := \begin{bmatrix} a_1^1 & \dots & a_1^N \\ a_2^1 & \dots & a_2^N \\ \vdots & & \vdots \\ a_{n_a}^1 & \dots & a_{n_a}^N \\ b_0^1 & \dots & b_0^N \\ \vdots & & \vdots \\ b_{n_b}^1 & \dots & b_{n_b}^N \end{bmatrix}. \quad (8)$$

We also define the *extended regressor* Ψ , which will be made up of past i/o data and parameter trajectories

$$\Psi_k := \phi_k \pi_k := \begin{bmatrix} -y_{k-1} \\ \vdots \\ -y_{k-n_a} \\ u_k \\ \vdots \\ u_{k-n_b} \end{bmatrix} [f_1(p_k) \quad \dots \quad f_N(p_k)], \quad (9)$$

where for notational simplicity we use a subscript for the time index (e.g. $p_k := p(k)$).

With the above definitions, it is easy to verify that the model (4)-(6) can now be rewritten as

$$y_k = \langle \Theta, \Psi_k \rangle, \quad (10)$$

where the inner product is the matrix inner product defined in the preliminaries.

In this paper, for the sake of simplicity we consider the case when no unmodeled dynamics are present and there exists a *true* parameter matrix Θ_o . We will also assume the measurement noise to be white and uncorrelated with the input. We list our assumptions for reference:

A1 No unmodeled dynamics.

A2 η_k is a white noise sequences such that

$$y_k = \langle \Theta, \Psi_k \rangle + \eta_k. \quad (11)$$

A3 η_k is uncorrelated with u_k

3 Identification of the LPV model

Let us consider the following loss function

$$J = J(\Theta) = \frac{1}{T} \sum_{k=0}^T E \{ \varepsilon(k, \Theta)^2 \}, \quad (12)$$

where the *prediction error* ε is defined as

$$\varepsilon(k, \Theta) = y_k - \langle \Theta, \Psi_k \rangle. \quad (13)$$

Let $\hat{\Theta}_k$ be the matrix of estimated parameters at time k . According to (Bamieh and Giarré, 1999) and (Bamieh and Giarré, 2000) the following algorithms can be derived.

LMS Algorithm: For the model in (11), under assumptions A.1, A.2 and A.3, the Least Mean Square identification algorithm is given by:

$$\begin{aligned}\Psi_k &= \phi_k \pi_k \\ \varepsilon_k &= y_k - \langle \hat{\Theta}_k, \Psi_k \rangle = y_k - \text{trace}(\hat{\Theta}_k^T \Psi_k) \\ \hat{\Theta}_{k+1} &= \hat{\Theta}_k + \alpha \varepsilon_k \Psi_k,\end{aligned}$$

where the extended regressor Ψ_k is defined by (9).

We note that all that is needed for the implementation of this LMS algorithm is the formation of the extended regressor, and taking the matrix inner product involved in forming ε_k .

For the Least Squares algorithm, we begin with some general remarks. Since (11) is in the form of a linear regression, an immediate method to solve the Least Squares problem is to “string out” the components of Ψ_k as a column vector, which then gives us the regression in its standard form. Instead, we find it very convenient to consider the Least Squares problem over a general finite dimensional inner product space, rather than the usual Euclidean space with its column vector notation.

RLS Algorithm: The Recursive Least Square (RLS) algorithm for the system (11) under assumptions A.1, A.2 and A.3 is given by

$$\begin{aligned}\varepsilon_k &= y_k - \langle \hat{\Theta}_{k-1}, \Psi_k \rangle = y_k - \text{trace}(\hat{\Theta}_{k-1}^T \Psi_k) \\ \hat{\Theta}_k &= \hat{\Theta}_{k-1} + K_k \varepsilon_k\end{aligned}\quad (14)$$

$$K_k = P_k \{ \Psi_k \} \quad (15)$$

$$P_k = P_{k-1} - P_{k-1} \frac{\Psi_k \otimes \Psi_k}{1 + \langle \Psi_k, P_{k-1} \Psi_k \rangle} P_{k-1} \quad (16)$$

3.1 Persistency of Excitation

For both the LMS and the RLS algorithm a *Persistency of Excitation* condition has to be satisfied in order to guarantee the consistency of the algorithm.

In (Bamieh and Giarrè, 2000) we proved the condition for the case of polynomial dependence of the coefficients on the parameters (i.e. when $f_l(p) = p^{l-1}$).

We report hereafter only an intuitively explanation.

Assuming that the input is sufficiently rich to insure that ϕ_k is PE in the above sense, what is needed is that the trajectory of p_k “visit” N distinct points infinitely many times. Our condition means that means that the rate at which p_k revisits each of these limit points should *not slow down*. Moreover, each of these revisits should be sufficient to ergodically extract the correlation data of ϕ_k .

The condition stated insures that enough data is gathered around the N interpolation points. The advantage of our schemes is that the set point does not have to vary slowly. For example, periodic parameter trajectories of any period (as long as N points are covered) will be sufficient for identification.

4 Example

Let us consider a simplified, lumped compression system model that describes rotating stall and surge instabilities:

$$\begin{aligned}\dot{\Omega} &= \frac{1}{l_c} (-\Xi + \Xi_C(\Omega) - 3R(\Omega - 1)) \\ \dot{\Xi} &= \frac{1}{4l_c B^2} (\Omega - \Omega_T) \\ \dot{R} &= 3\mu R(2\Omega - \Omega^2 - R)\end{aligned}\quad (17)$$

where the time-dependence is omitted for sake of simplicity, the variable Ω is the compressor mass flow, the variable Ξ is the pressure rise of the compressor and the variable R is the amplitude square of the first harmonic mode of the rotating stall disturbance, Ω_T is the input, the throttle mass flow. Hereafter, we set $l_c = 1$, $\mu = 0.6$. This is based on the Moore and Greitzer model ((Moore and Greitzer, 1986)).

The objective of rotating stall and surge control is to achieve the maximum compression efficiency while eliminating stall, assuming the state variable available for feedback. In order to design a gain scheduling control, see (Zappa *et al.*, 2001), we need to model the compression system dynamics into an LPV form. The identification of the LPV model form data is extensively reported in (Bamieh *et al.*, 2001).

According to (Tu and Shamma, 1998), we choose $\omega(k) = \Omega(k) - 2$, $\omega_T(k) = \Omega_T(k) - 2$, and the variable $p(k) = \omega(k)$ as the *scheduling* variable. Then, we get the following relaxed Quasi-LPV model, using the relaxation auxiliary variable $v(k)$:

$$\begin{aligned}\mathbf{x}(k+1) &= A(p(k))\mathbf{x}(k) + B_1 u(k) + B_2 d(k) \\ \mathbf{y}(k) &= C(p(k))\mathbf{x}(k)\end{aligned}\quad (18)$$

where the status variable is

$$\mathbf{x}(k) = [v(k) \quad \xi(p(k)) - \xi_{eq}(p(k)) \quad R(k)]',$$

$d(k) = 0.75 \sin(2p(k)) + 0.25 \sin(14p(k))$ takes into account the compressor characteristic uncertainties, $u(k) = \omega_T(k) - \omega_{T_{eq}}(k)$ is the input and the matrices are given by:

$$A(p(k), \rho(k)) = \begin{bmatrix} 1 & -\frac{T}{l_c} & -\frac{3T}{l_c}(p+1) \\ 0 & 1 - \frac{T}{l_c}(3p + \frac{3}{2}p^2) & -\frac{3T}{l_c}(3p + \frac{3}{2}p^2)(p+1) \\ 0 & 0 & 1 - 3\mu T(2p + p^2) - 0.9\mu T\rho \end{bmatrix}\quad (19)$$

where $\rho = \rho(k)$ is a disturbance taking into account some non-linear terms, and $p = p(k)$ is the scheduling parameters,

$$B_1 = \begin{bmatrix} 0 \\ -\frac{T}{4l_c B^2} \\ 0 \end{bmatrix}, \quad (20)$$

$$B_2 = \begin{bmatrix} 0.2T \\ 0 \\ 0 \end{bmatrix}, \quad (21)$$

and $C = I$, $D = 0$,

The sampling time T is chosen equal to $0.04s$.

The constraints to be satisfied by the control procedure are $|p(k)| \leq 0.5$, $|p(k+1) - p(k)| \leq T \cdot 0.05$, $|v(k)| \leq 0.5$, $|v(k+1) - v(k)| \leq T$.

In order to identify the model, the system has to be first controlled in order to stabilize the plant with a feedback control of the form $u(t) = [k_1 \ k_2 \ k_3]x(t) + w(t)$.

A simple feedback control is obtained solving a LQ optimal control, considering a finite set of possible values for the parameters. Let F_i be the feedback corresponding to $p(k) \in P_i$: $\frac{p_i - p_{i-1}}{2} \leq p(k) \leq \frac{p_{i+1} - p_i}{2}$. In particular we have chosen the set of possible values as follows : $\{-0.15, -0.13, -0.11, \dots, -0.01, 0.01, 0.03, \dots, 0.15\}$. In the following we are considering only the surge control problem, setting $R = 0$.

In open loop, the above system can be modeled as

$$A(\delta, p)y(k) = B(\delta, p)u(k) + B_d(\delta, p)d(k)$$

with

$$A(\delta, p) = 1 + A_1(p)\delta^{-1} + A_2(p)\delta^{-2}$$

and for $i = 1, 2$, $A_i(p) = a_i^1 + a_i^2 p + a_i^3 p^2$ and

$$B(\delta, p) = \begin{bmatrix} B_1(\delta, p) \\ B_2(\delta, p) \\ 0 \end{bmatrix}$$

where $B_1(\delta, p) = \beta_{12}(p)\delta^{-2} + \beta_{13}(p)\delta^{-3}$,

$B_2(\delta, p) = \beta_{21}(p)\delta^{-1} + \beta_{22}(p)\delta^{-2} + \beta_{23}(p)\delta^{-3}$

and

$$B_d(\delta, p) = \begin{bmatrix} \beta_{d11}\delta + \beta_{d12}\delta^{-2} + \beta_{d13}\delta^{-3} \\ 0 \\ 0 \end{bmatrix}$$

The LMS algorithm is used with $\alpha = 20$ and input u and scheduling parameters p are chosen in order to satisfy the persistency condition and a simulation of $N = 4000$ steps is performed.

On line, from the estimated value of the closed loop parameters, the open loop ones are determined using the knowledge of the assumed value of p and the corresponding feedback gain.

The obtained identified final value of the matrix $\hat{A}(p)$ is

$$\hat{A}(p) = \begin{bmatrix} -1.99 & 0.11 & 0.06 \\ 0.99 & -0.12 & -0.06 \end{bmatrix}$$

when the true matrix of the LPV system calculated off-line is

$$A(p) = \begin{bmatrix} -2 & 0.12 & 0.06 \\ 1 & -0.12 & -0.06 \end{bmatrix}$$

In Fig. 1 the estimated parameters time plot of the $\hat{A}(p)$ coefficients are compared with the off-line evaluated coefficients.

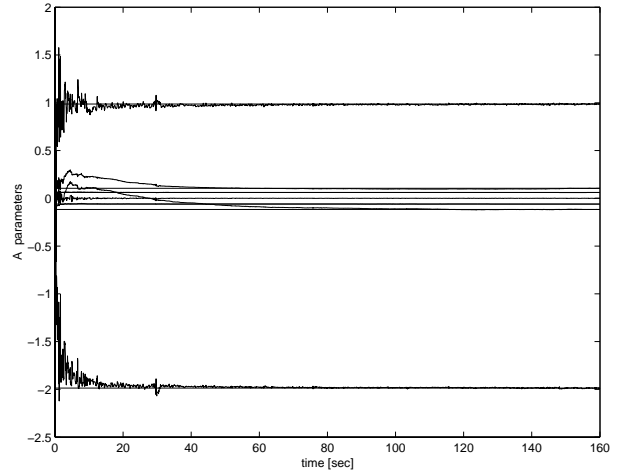


Figure 1: LMS algorithm: time plot of the comparison of estimated and off-line evaluated $A(p)$ -parameters.

5 Conclusions

An overview of LPV models for gain scheduling control and identification has been presented.

We have considered Least Squares type identification problems for LPV systems with polynomial dependence on the parameters. For this particular class, we have shown that the problem can be solved by linear regression. By representing the regression as a matrix inner product, we derived simple representation of the RLS and LMS algorithms, and conditions for persistency of excitation.

The identification procedure is applied to the model of compressors for jet engines. The model is controlled in order to avoid rotating stall and surge. Once the LPV model based on the Moore and Greitzer nonlinear model of compressors is identified, the design of a robust gain scheduling predictive controller will be implemented.

References

- Bamieh, B. and L. Giarrè (1999). Identification for linear parameter varying models. *In Proc. of the 38th IEEE Control and Decision Conference*.
- Bamieh, B. and L. Giarrè (2000). Identification for a general class of lpv models. *In Proc. of the IFAC System Identification*.
- Bamieh, B., L. Giarrè, T. Raimondi, D. Bauso, M. Lodato and D. Rosa (2001). Lpv model identification for the stall and surge control of jet engine. *In Proce. of the 15th IFAC Symposium on Automatic Control in Aerospace*.
- Blanchini, F. (1994). Ultimate boundedness control for discrete-time uncertain system via set-induced lyapunov function. *IEEE Trans. on Automatic Control* **39**(2), 428–433.

- Blanchini, F. (1999). The gain scheduling and the robust state feedback stabilization problems. *Proc. of the IEEE Conf. on Decision and Control*.
- Campi, M.C. (1994). Exponentially weighted least squares identification of time-varying systems with white disturbances. *IEEE Trans. on Signal Processing* **42**, 2906–2914.
- Lee, L. and K. Poolla (1999). Identification of linear parameter-varying systems using nonlinear programming. *ASME Journal of Dynamic Systems, Measurement, and Control* **121**, 71–78.
- Mazzaro, M.C., B.A. Movsichoff and R. Sanchez Pe na (1999). Robust identification of linear parameter varying systems. *In Proc. of the American Control Conference*.
- Moore, F. and E. Greitzer (1986). A theory of post-stall transients in axial compression systems: Part i - development of equations. *Journal of Engineering for Gas Turbines and Power* (108), 68–76.
- Nemani, M., R. Ravikanth and B. Bamieh (1995). Identification of linear parametrically varying systems. *Proc. of the 34th IEEE Control and Decision Conference*.
- Packard, A. (1994). Gain scheduling via linear fractional transformations. *System and Control Letters* **22**, 79–92.
- Previdi, F. and M. Lovera (1999). Identification of a class of linear models with nonlinearly varying parameters. *Proc. of the V European Control Conference*.
- Rugh, W.J. and J.S. Shamma (2000). Research on gain scheduling. *Automatica* **36**, 1401–1425.
- Shamma, J. (1996). Linearization and gain scheduling. In: *Control Handbook* (W. Levine, Ed.). pp. 388–396. CRC Press.
- Shamma, J. and D. Xiong (1999). Set-valued methods for linear parameter varying systems. *Automatica* **35**, 1081–1089.
- Sznaier, M., M.C. Mazzaro and T. Inanc (2000). An lmi approach to control oriented identification of lpv systems. *In Proc. of American Control Conference*.
- Tu, K.H. and J.S. Shamma (1998). Nonlinear gain-scheduled control design using set-valued methods. *In Proc. of American Control Conference*.
- Zappa, G., P. Falugi, L. Chisci and L. Giarré (2001). Lpv models based predictive control of the stall and surge for jet engine. *In Proc. of IFAC workshop on Periodic Control Systems*.