

**M.S. YALIN MEMORIAL Mini-Colloquium on Fluvial Eco-Hydraulics  
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**THE VON KÁRMÁN CONSTANT FOR FLOWS OVER MOBILE BEDS. LESSONS  
LEARNED FROM DIMENSIONAL ANALYSIS AND SIMILARITY**

Rui M. L. Ferreira

CEHIDRO, Instituto Superior Técnico, ULisbon, Portugal,

*e-mail: ruif@civil.ist.utl.pt*

The nature of von Kármán constant ( $\kappa$ ), i.e. universal, Re-independent or, in general, flow dependent, has been subjected to considerable investigation. Recently, it has been proposed that, in flows over hydraulically rough mobile boundaries, the value of  $\kappa$  is influenced by bedload movement (e.g. Gaudio et al. 2010). However, Ferreira et al. 2012 argued that, under those conditions, variations in the value of  $\kappa$  can be explained by the positioning of the zero of the log-law. Rather than advocating a particular view on the nature of  $\kappa$ , the objective of this work is to bring to light the general form of  $\kappa$  as a function of bed, flow, namely turbulence, and fluid parameters. This procedure operates a shift in the argumentation: the nature of  $\kappa$  follows from the nature of the parameters involved in the functional relation. The quantification of these avoids the ambiguities involved in the quantification of  $\kappa$  from the vertical profile of the longitudinal velocity. To develop the functional relation of  $\kappa$  similarity arguments leading to the log-law are employed and VaschyBuckingham's theorem is applied (Lo, L'vov, Pomyalov, and Procaccia 2005). A new key step is the assumption that the shear-rate is a two-phase variable, depending on flow as well as sediment parameters. This step follows Yalin's 1971 methodology for hydraulic similarity and Yalin's 1977 formalization of similarity analysis for two-phase flow variables. Introducing appropriate hypothesis concerning complete and incomplete similarity, the following non-dimensional relation is obtained

$$\kappa = \frac{1}{8} \left( (-b_{13} / P_w) C_{2w} \right)^{3/2}$$

where  $-b_{13}$  is the longitudinal-lateral component of the anisotropy tensor,  $P_w$  is the fraction of TKE associated to vertical motion and  $C_{2w}$  is the constant in the 2<sup>nd</sup> order structure function. These parameters should be evaluated in the logarithmic layer. Hence, the argument on the universality of the von Kármán constant becomes an investigation on the structure of turbulence in flows over mobile beds. In particular, changes in turbulent anisotropy are susceptible to justify boundary-dependent values of  $\kappa$ .

Laboratory results concerning flows over porous immobile and mobile beds are presented. The data is not supportive of fundamental changes in the structure of turbulence for flows over weakly mobile gravel-sand beds. Research supported by the Portuguese Foundation for Science and

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