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DOTTORATO DI RICERCA IN SCIENZE AGRARIE, FORESTALI E AMBIENTALI
INDIRIZZO IDRONOMIA AMBIENTALE, XXIX CICLO

ESTIMATING RAINFALL EROSIVITY BY DROP SIZE DISTRIBUTION

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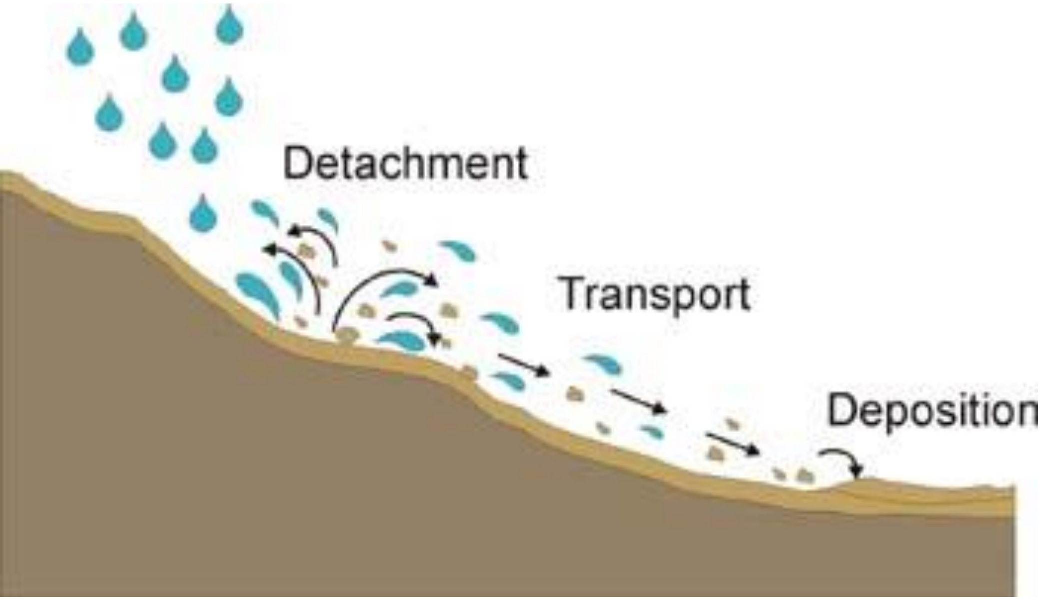
1.Introduction

The water erosion is the main cause of the modeling of extended portions of the earth's surface.

Soil erosion processes include the *detachment* of soil particles, their *entrainment* in the water flow, *transport* and *deposition* of the sediments



These processes are related to all rainfall characteristics defining

RAINFALL EROSIVITY

can be represented by its kinetic energy per unit time and area, named

KINETIC POWER (Pn)

1.Introduction

P_n/I = kinetic energy of the unit volume of rainfall

Wischmeier and Smith (1978)

$$\frac{P_n}{I} = (11{,}87 + 8{,}73 \log I) \quad I \le I_t$$
$$\frac{P_n}{I} = (11{,}87 + 8{,}73 \log I_t) \quad I > I_t$$

I_t = 76 mm/h

$[P_n]$ = J m^{-2} h^{-1}

They justified this trend taking into account that median volume diameter, D_0, (i.e. the diameter that divides the DSD in two parts of equal volume) does not more increase when $I > I_t$

Kinnell (1981)

$$\frac{P_n}{I} = a\left(1 - b \exp(-c\, I)\right)$$

According to:

- Brown and Foster (1987)

a =29 J m^{-2} mm^{-1} b = 0.72 c = 0.05 h mm^{-1}

- McGregor et al. (1995)

a =29 J m^{-2} mm^{-1} b = 0.72 c = 0.082 h mm^{-1}

1.Introduction

P_n can be also determined by adding the contribution of single raindrops once their mass and terminal velocity are known. In other words, detachability of soil due to a rainfall event can be indirectly determined by

- *Drop Size Distribution*, DSD, (*N(D) dD*)

- Rain drop terminal velocity (*V*)

$$P_n = 10^{-6}\frac{\rho\pi}{12}\int_0^{\infty} V^2 D^3 N(D) dD$$

1.Introduction

For hydrological aims, the *DSD* usually refers to the number of droplets *N(D)dD*, having diameter between D and *D+dD*, that reach a unit horizontal area during a unit time. One of the most applied drop size distribution, because of its flexibility and its applicability in different climatic conditions, is the gamma distribution of Ulbrich (1983):

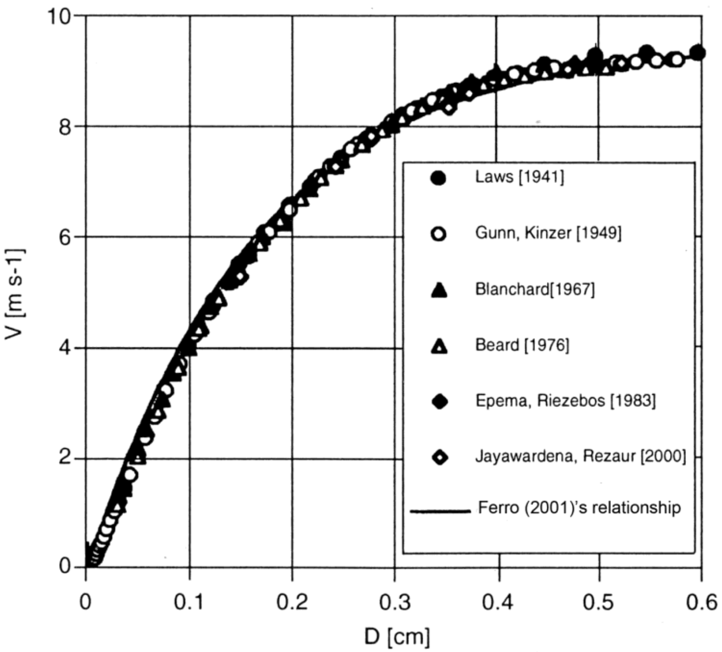
$$N(D)\,dD = N_0\, D^{\mu}\, exp(-\Lambda\, D)\, dD$$

μ, Λ, N_o = distribution parameters.

For natural precipitation the terminal velocity (*V*) of the raindrop depends only on its diameter (*D*). For estimating *V* of the drop having diameter *D* (cm), Ferro (2001) proposed:

$$V = V_{max}\left[1 - \exp(-a_n D)\right]$$

V_{max} = 9.5 m s^{-1} a_n = 6 cm



1.Introduction

Using both the Ulbrich's distribution and the relationship proposed by Ferro (2001), the following relationship can be theoretically deduced (Carollo and Ferro, 2015) :

$$P_n = 10^{-6}\frac{\rho\pi}{12}\int_0^\infty V^2 D^3 N(D)dD$$

$$\frac{P_n}{I} = 10^{-6}\frac{9.5^2}{7.2}\rho\left[1-\frac{2\Lambda^{4+\mu}}{(6+\Lambda)^{4+\mu}}+\frac{\Lambda^{4+\mu}}{(12+\Lambda)^{4+\mu}}\right]$$

$[P_n]$ = W m^{-2}

P_n can be determined if *I* and μ and Λ parameters are known

1.Introduction

Considering Ulbrich's distribution, the probability *P*(*D*) that raindrop diameter is less than *D* can be calculated as (Carollo and Ferro, 2015):

$$P(D) = \frac{\Lambda^{\mu+1}}{\Gamma(\mu+1)} \int_0^D D^{\mu} \exp(-\Lambda D)\, dD$$

Γ = gamma function

P(D) depends only on μ and Λ parameters.

By fitting *P*(*D*) to drop-size empirical frequency distribution, we estimate μ and Λ parameters.

1.Introduction

Parameter estimate methods:

- *Maximum likelihood method (ML)*
- *Momentum method* (MM)

$$D_{50} = \frac{0.67 + \mu}{\Lambda} \qquad D_0 = \frac{3.67 + \mu}{\Lambda}$$

$$\mu = D_0 \Lambda - 3.67 \qquad \Lambda = \frac{3}{D_0 - D_{50}}$$

(Ulbrich, 1983)

D_{50} = median drop diameter
D_0 = median volume drop diameter

2.Objectives

- ✓To analyze DSD of the rainfalls detected at El Teularet (Spain) and Palermo (Italy).
- ✓ To verify the reliability of the Ulbrich's distribution to DSD detected in both sites.
- ✓ To verify the reliability of the kinetic power relationships proposed by Wischmeier and Smith (1978) and Kinnell (1981) for the precipitations registered at Palermo and El Teularet.
- ✓ To compare P_n/I, D_0 measurements of the present investigation with the ones detected in other climate and environmental contests.

Experimental sites

«El Teularet» experimental station

Sierra de Enguera at 100 km southwest from Valencia

Altitude: 760 m a.s.l.

Climate:

- Mediterranean continental
- Cool winter and hot summer with rainfall irregularly distributed during the year
- BSk cold semi-arid (Köppen climate classification)

Palermo experimental station

Altitude: 40 m a.s.l.

Climate:

•Mediterranean temperate

•Dry and hot summer and mild and rainy winter

•CSa dry-summer subtropical (Köppen climate classification)

2. Experimental equipment

Optical disdrometer (ODM 470 – Eigenbrodt)

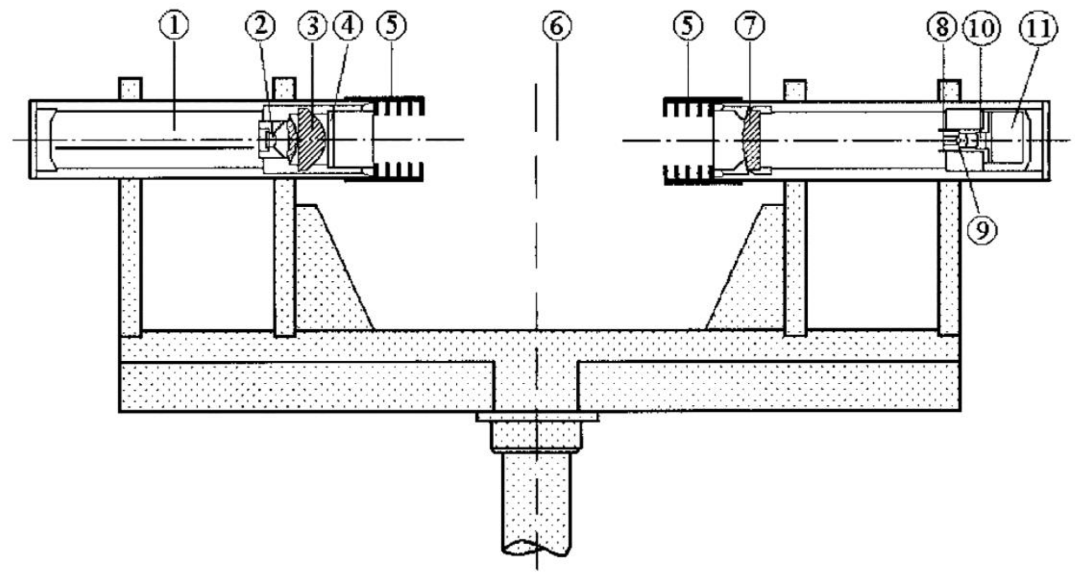
For each **rainfall minute**

- Measures drop diameters in the range 0.035 - 0.60 cm.
- Divides diameter range into 128 classes having width equal to 0.05 mm.
- Gives as output the number of the rain drops belonging to each class.

2. Experimental equipment

Drop diameter is measured registering light damping due to the passage of the drop in the control cylindrical volume (120 mm length and 22 mm diameter) between two diodes.

Without rainfall the diode produces a voltage of 5 volt.



1) electronics, 2) light-emitting diode, 3) lens system, 4) window, 5) baffles, 6) sensitive volume, 7) achromatic collector lens, 8) optical blend, 9) ocular, 10) photo diode, and 11) electronics compartment.

The passage of a drop determines, instead, a light dumping and a consequence voltage reduction proportional to drop diameter.

2. Experimental equipment

A rainfall detector (model IRSS88) placed near the disdrometer, signals rainfall occurrence (at least 5 drops in 90 s) and so switches on disdrometer. After 60s without rainfall it switches off the disdrometer.

These two instruments are connected to a computer, that permits to record data

3. Results and discussions

In order to exclude both rainfalls having a low erosive power and drop size distributions having a small sample size, the DSD analysis was only developed for:

- $I \geq 0.5$ mm/h
- Measured diameter classes were at least 20

Palermo

544 rainfall events
45802 single DSD

June 2006- April 2014

I = 0.5 ÷ 203 mm/h

El Teularet

79 rainfall events
5537 single DSD

July 2015 - May 2016

I = 0.5 ÷ 150 mm/h

3. Results and discussions

To analyze the influence of rainfall intensity the 45802 DSD detected at Palermo and the 5537 DSD detected at El Teularet were aggregated for intensity classes differing in width:

- 1 mm/h $I < 30$ mm/h
- 2 mm/h $30 < I < 50$ mm/h
- 5 mm/h $50 < I < 100$ mm/h
- 10 mm/h $I > 100$ mm/h

Obtaining:

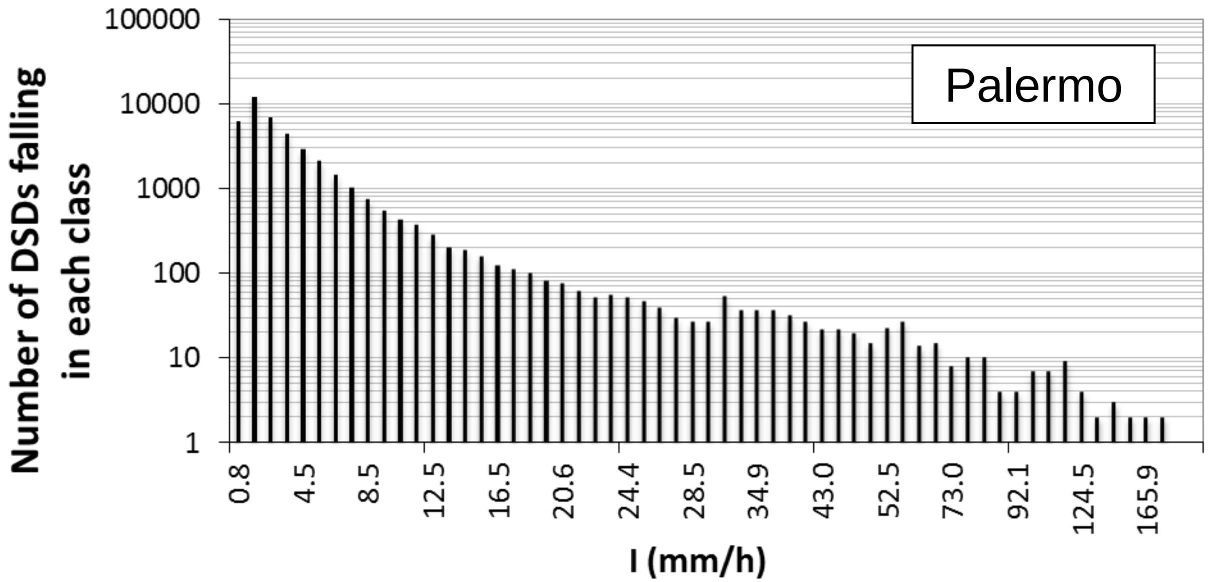
- ***59 DSDs*** ***0.8 < I < 203 mm/h (Palermo)***
- ***54 DSDs*** ***0.7 < I < 150 mm/h (El Teularet)***

3. Results and discussions

the number of DSDs falling in each class is < 10 for

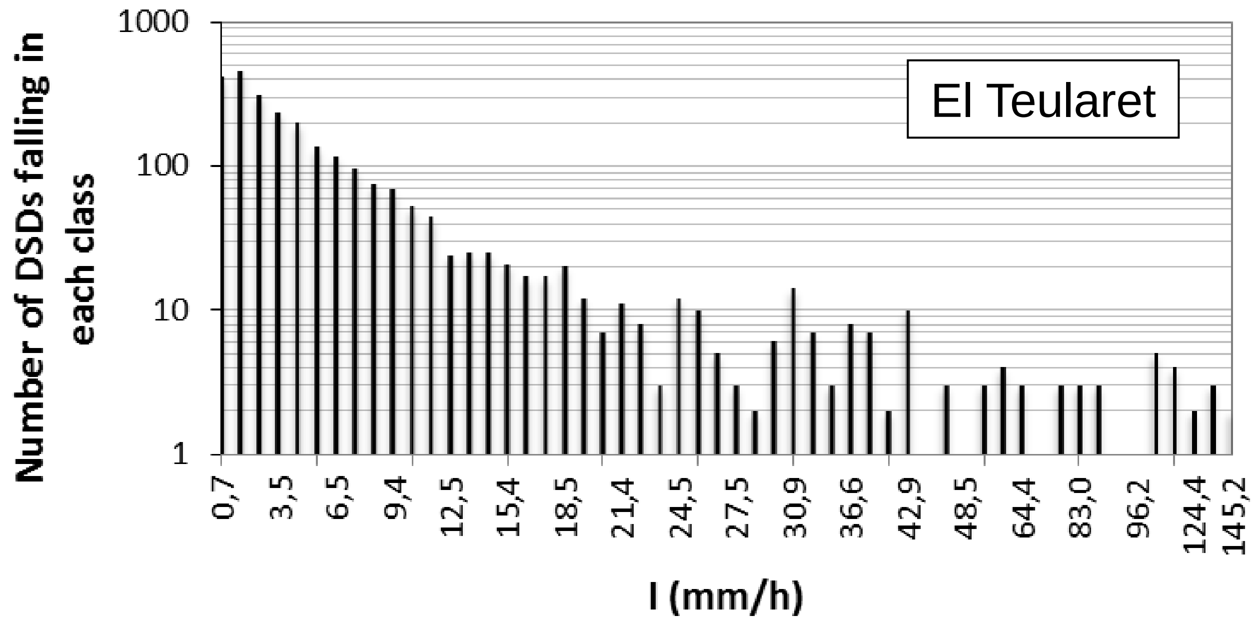
I >85 mm/h

(Palermo data)



I > 30 mm/h

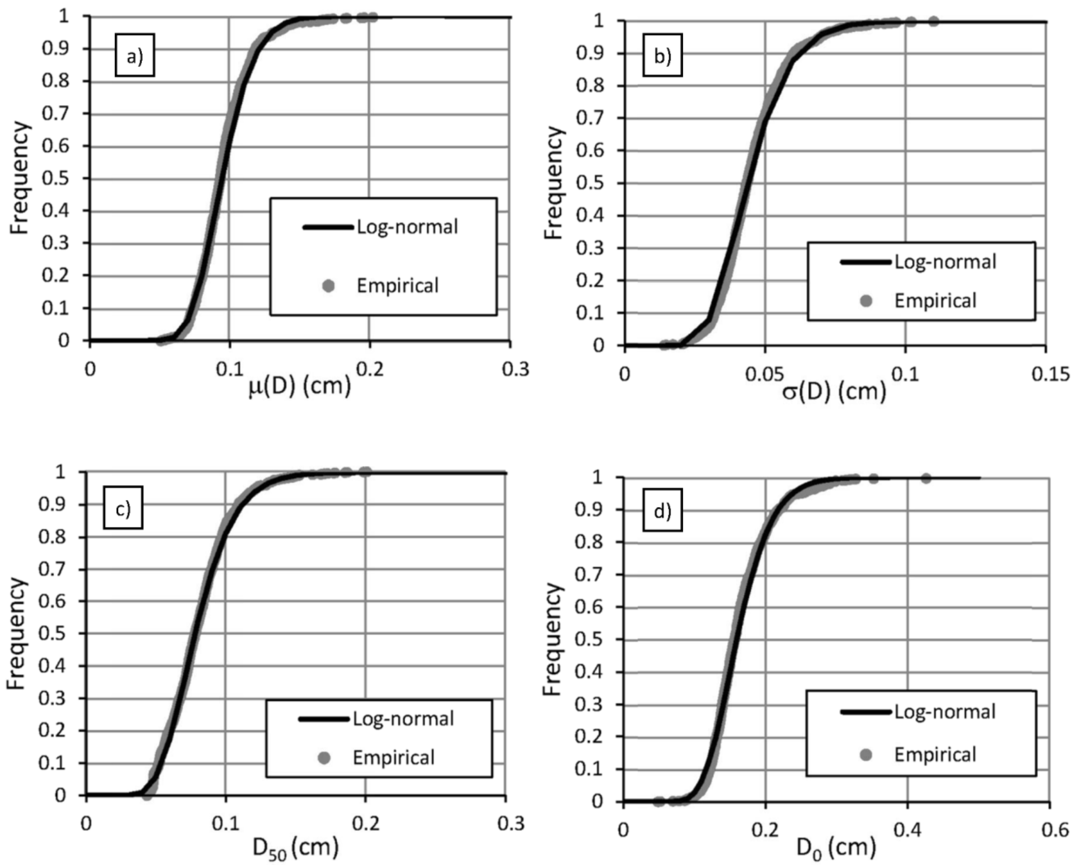
(El Teularet data)



3. Results and discussions

Test the hypotesis that DSD can be aggregated

(Palermo DSD)

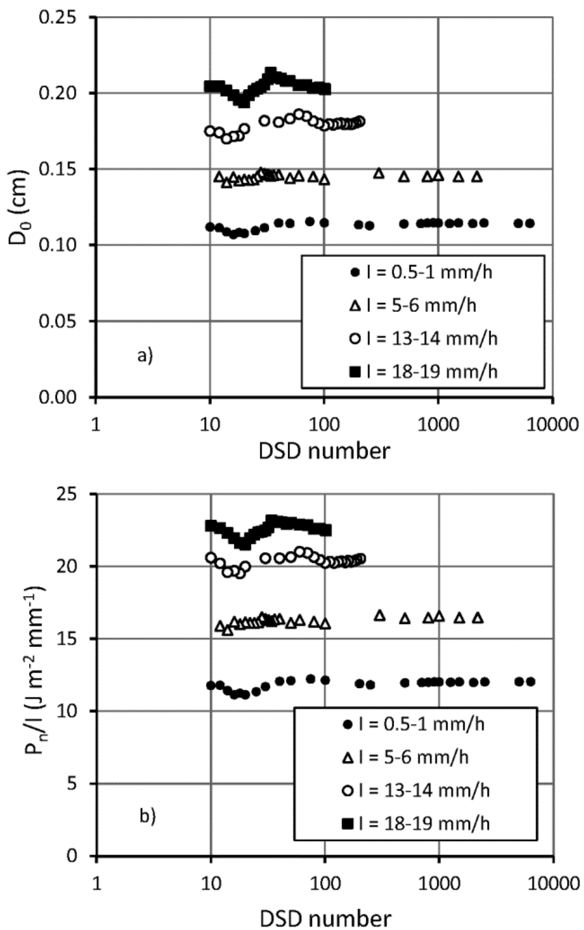


For each intensity class D_0, D_{50}, $\mu(D)$ and $\sigma(D)$ of each DSD were considered to determine their empirical frequency distribution.

The D_0, D_{50}, $\mu(D)$ and $\sigma(D)$ empirical frequency distribution agree with lognormal distribution.

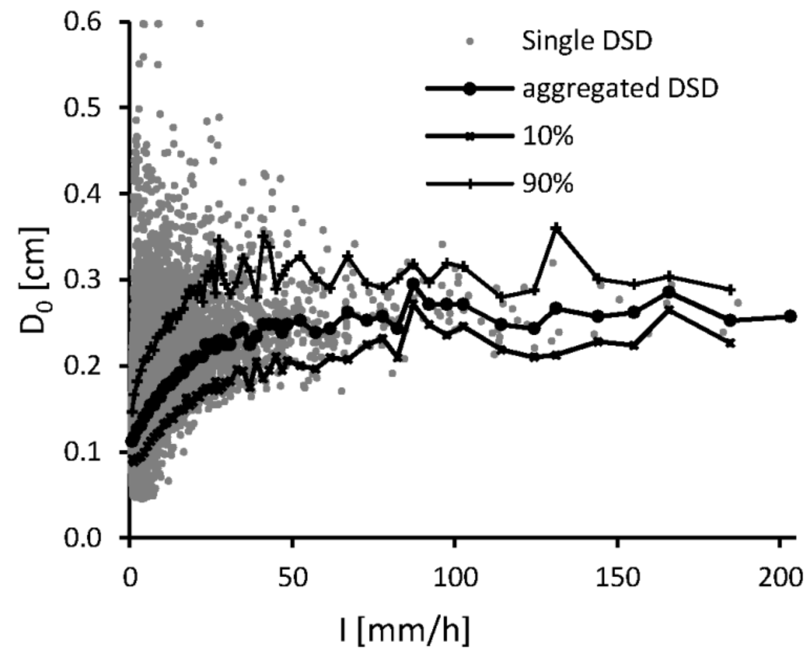
The values of D_0, D_{50}, $\mu(D)$ and $\sigma(D)$ are identically distributed within a fixed rainfall intensity class

3. Results and discussions



The decreasing number of the DSD until to 10 units does not significantly affects the values of the two rainfall energy variables

Relationship between both the median volume diameter and rainfall intensity for single and aggregated DSDs

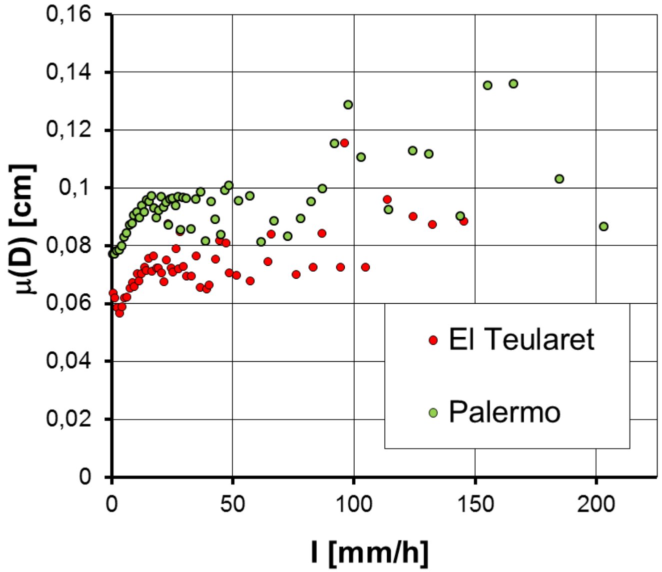


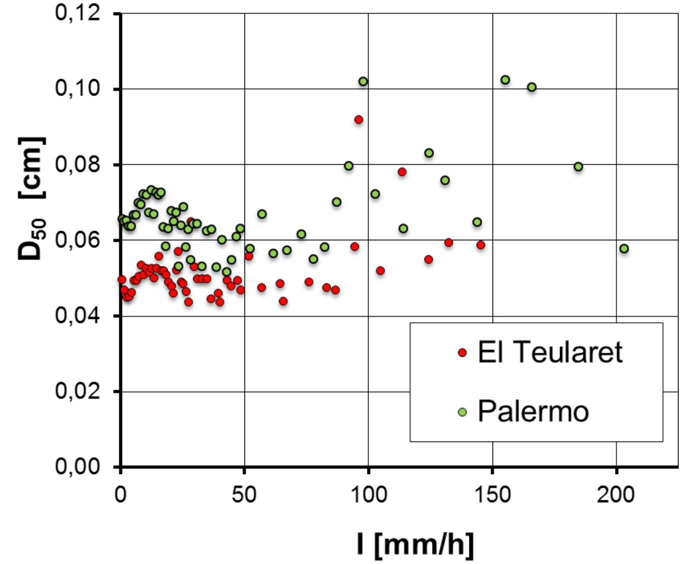
(I, D_0) pairs falling out the confidence interval and corresponding to single DSDs characterized by low rainfall intensities, can be also far from the upper bounding curve. On the contrary, for rainfall intensity greater than 70 mm/h, the (I, D_0) pairs of single DSDs are always near to the line representative of the aggregated DSD.

3. Results and discussions

For a given rainfall intensity

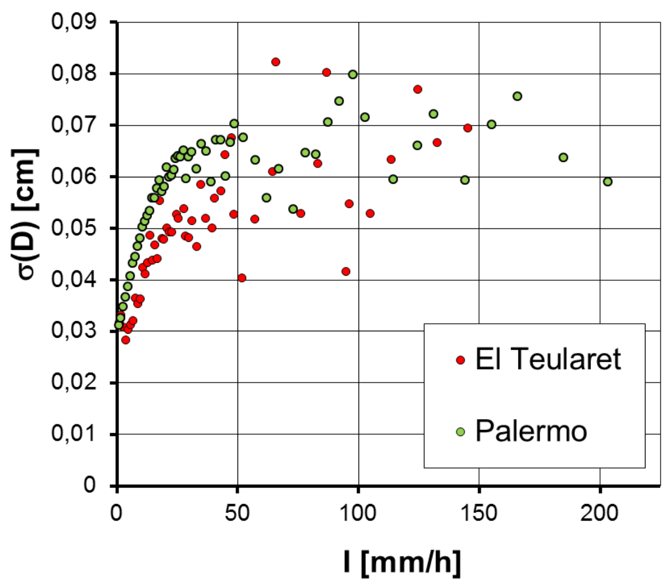
Higher values of $\mu(D)$, D_{50} and $\sigma(D)$ detected at Palermo

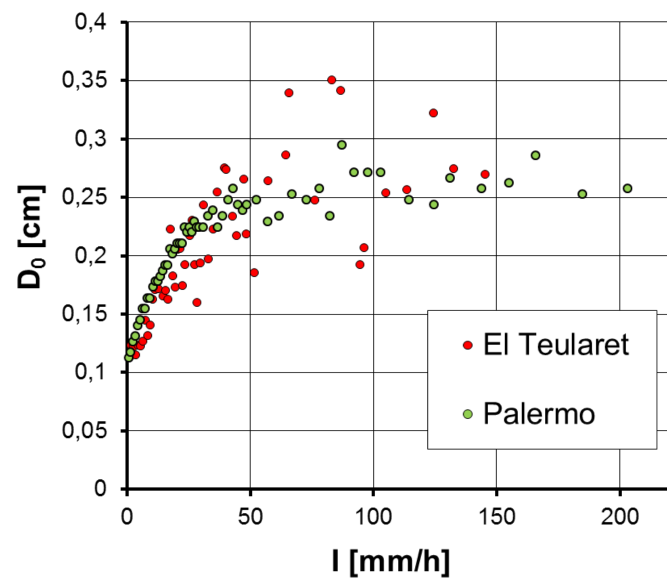




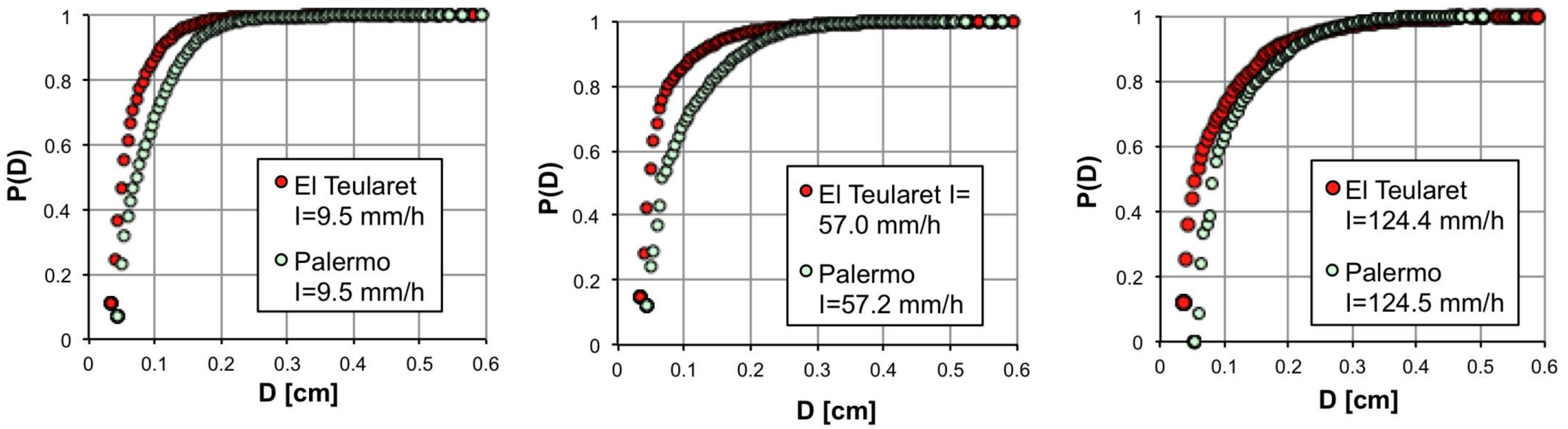
For both datasets

- $\mu(D)$ increases with I for I<15 mm/h no trend for I>15 mm/h
- no relation D_{50} - I
- $\sigma(D)$ and D_0 increase with I for I<40 mm/h no trend for I>40 mm/h





3. Results and discussions



for a fixed rainfall intensity

- the Palermo DSD is different from the one detected at El Teularet
- The Palermo DSD tends to move more towards right being characterized by greater values of D_{50} and $\sigma(D)$.

3. Results and discussions

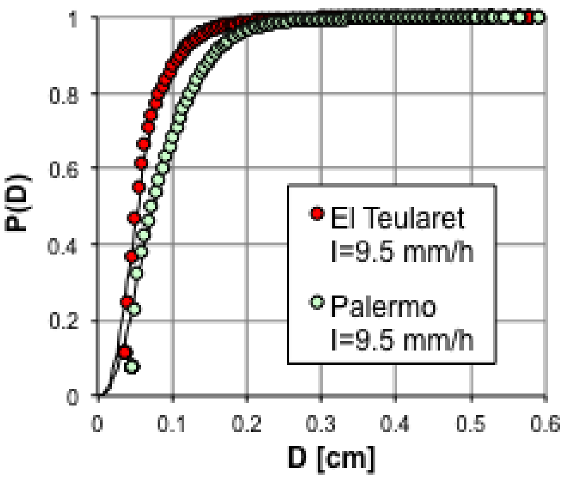
Fitting of Ulbrich's distribution (μ, Λ)

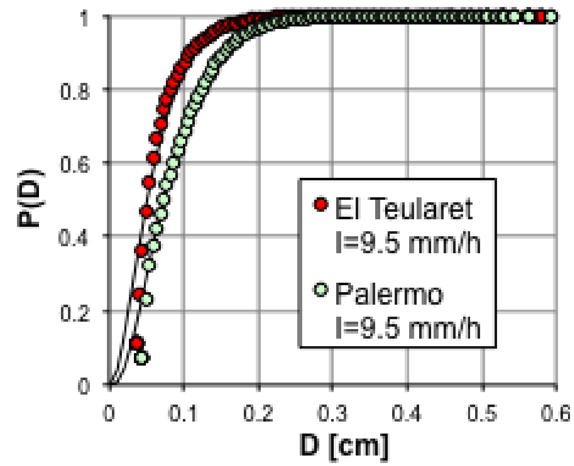
$$P(D) = \frac{\Lambda^{\mu+1}}{\Gamma(\mu+1)} \int_0^D D^{\mu} \exp(-\Lambda D)\, dD$$

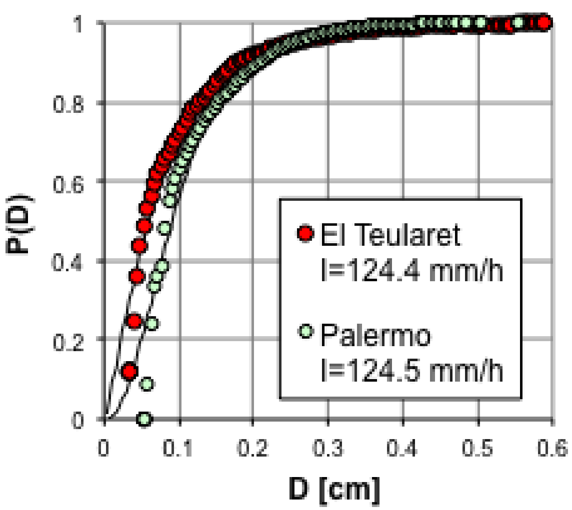
Maximum likelihood method (ML)

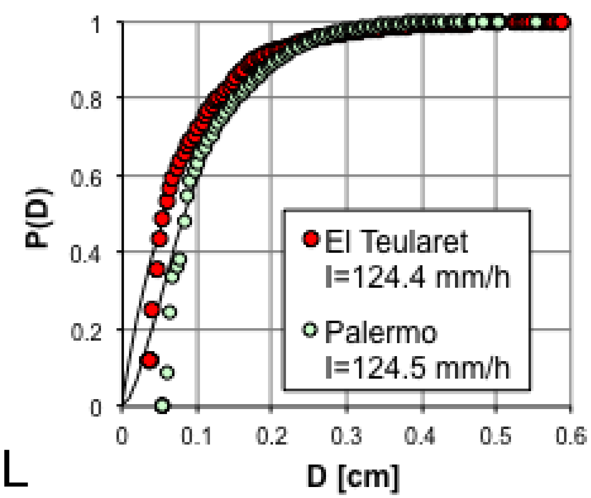
Momentum method(MM)

setting theoretical values of D_0 and D_{50} equal to measured ones





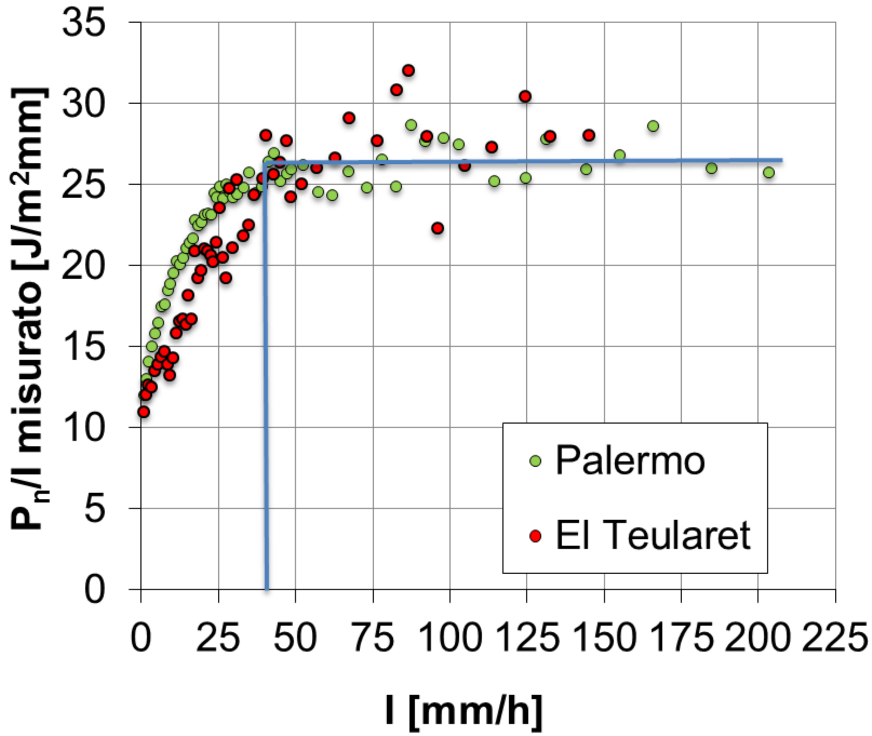




best fitting by ML

3. Results and discussions

The aggregated DSDs were used to determine the experimental rainfall kinetic power (measured P_n)



I_t= 40 mm/h

- For I<30 mm/h El Teularet presents P_n/I values lower than Palermo one
- For I> 30 mm/h the pairs (I, P_n/I) detected at Palermo and El Teularet are more overlapped.

  The same rainfall intensity determines precipitation characterized by different DSDs and similar energetic characteristics.

- For I < 40 mm/h, P_n/I increases with I
- For I $\geq$ 40 mm/h, assumes a quasi constant value
- The measurements agree the Wischmeier and Smith approach even if a lower threshold value (I_t= 40 mm/h) was observed.

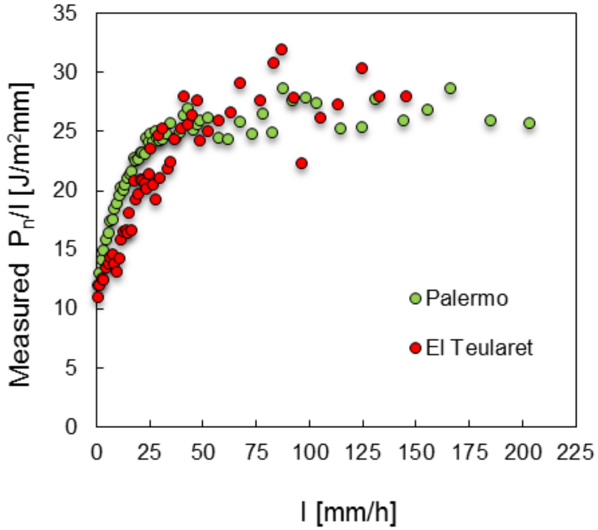
P_n/I = kinetic energy of the unit volume of rainfall

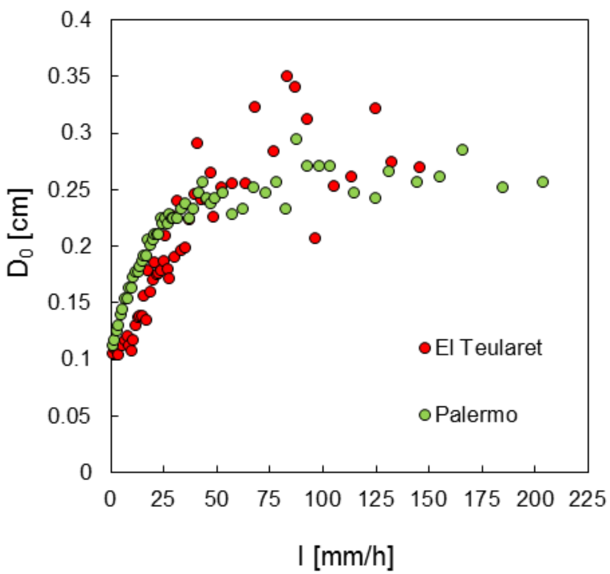
3. Results and discussions

This trend can be justified taking into account the phenomena of aggregation (collapse) and disaggregation of the raindrops.

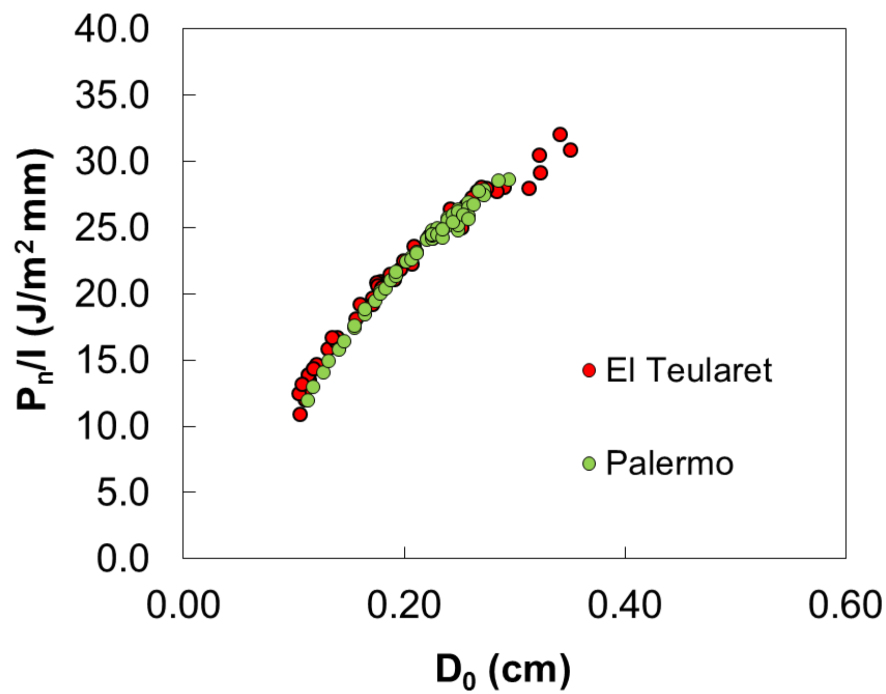
- for $I \leq I_t$, an increase of rainfall intensity determines an increase of the size of the drops because the collapse effects prevail on the disaggregation ones. The increase of dropsize yields an increase of terminal velocity and, as consequence, an increase of kinetic energy of the rainfall unit volume.
- for $I > I_t$, an increase of rainfall intensity determine only an increase of number of raindrops that reach the soil without varying the dropsize distribution. Probably, an equilibrium between aggregation and disaggregation phenomena occurs.

3. Results and discussions





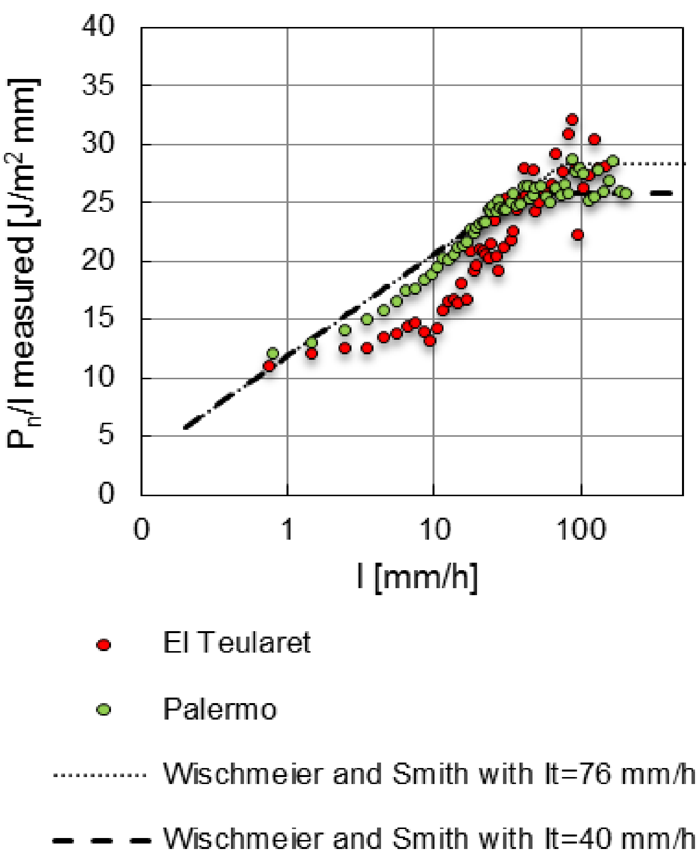
- The relation P_n/I – I is similar to D_0 – I one because P_n/I depends on D_0



- The pairs (D_0, P_n/I) relative to the two datasets are overlapped
- The measurements agree with hypothesis of Wischmeier and Smith

3. Results and discussions

Reliability of Wischmeier and Smith (1978) relationship

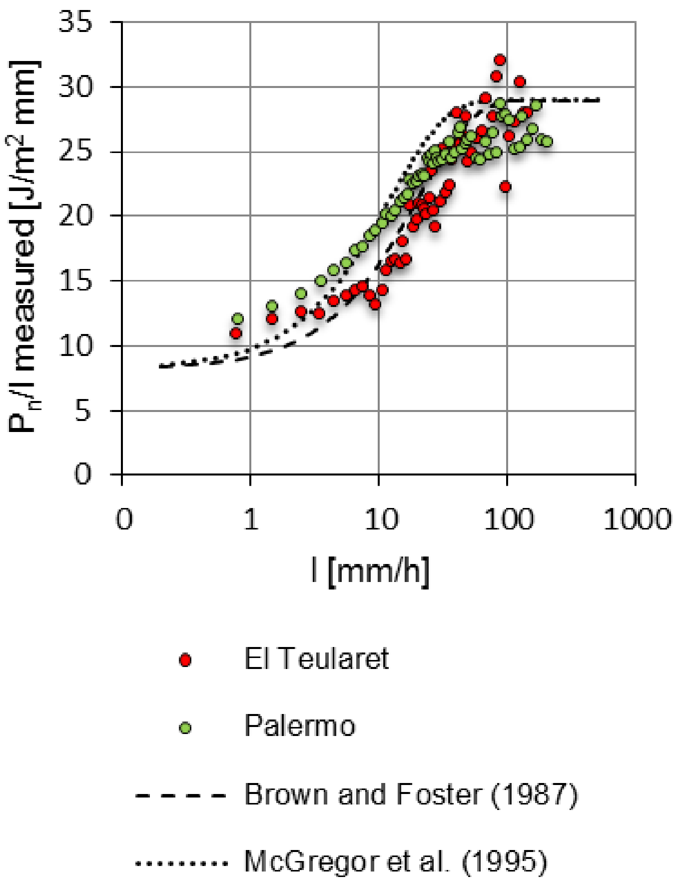


setting $I_t = 40$ mm/h

- Palermo
  - For 2 < *I* < 20 mm/h slight overestimation
  - For *I* > 40 mm/h slight overestimation
- El Teularet
  - For 20 < I <40 mm h^{-1} is able to reproduce the measured P_n/I values
  - For 2 < *I* < 20 mm/h systematic overestimation
  - for *I* > 40 mm/h slight overestimation

3. Results and discussions

Reliability of Kinnell (1981) relationship



The mathematical shape of Kinnell's relationship describes the observed trend better than the Wischmeier and Smith one.

It needs to be calibrated

Palermo

✓ Brown and Foster (1987):

- for *I* < 20 mm/h systematic underestimation
- for *I* > 30 mm h^{-1} slight overestimation

✓ McGregor et al. (1995):

- for *I* < 7 mm/h underestimation
- for *I* > 10 mm h^{-1} systematic overestimation

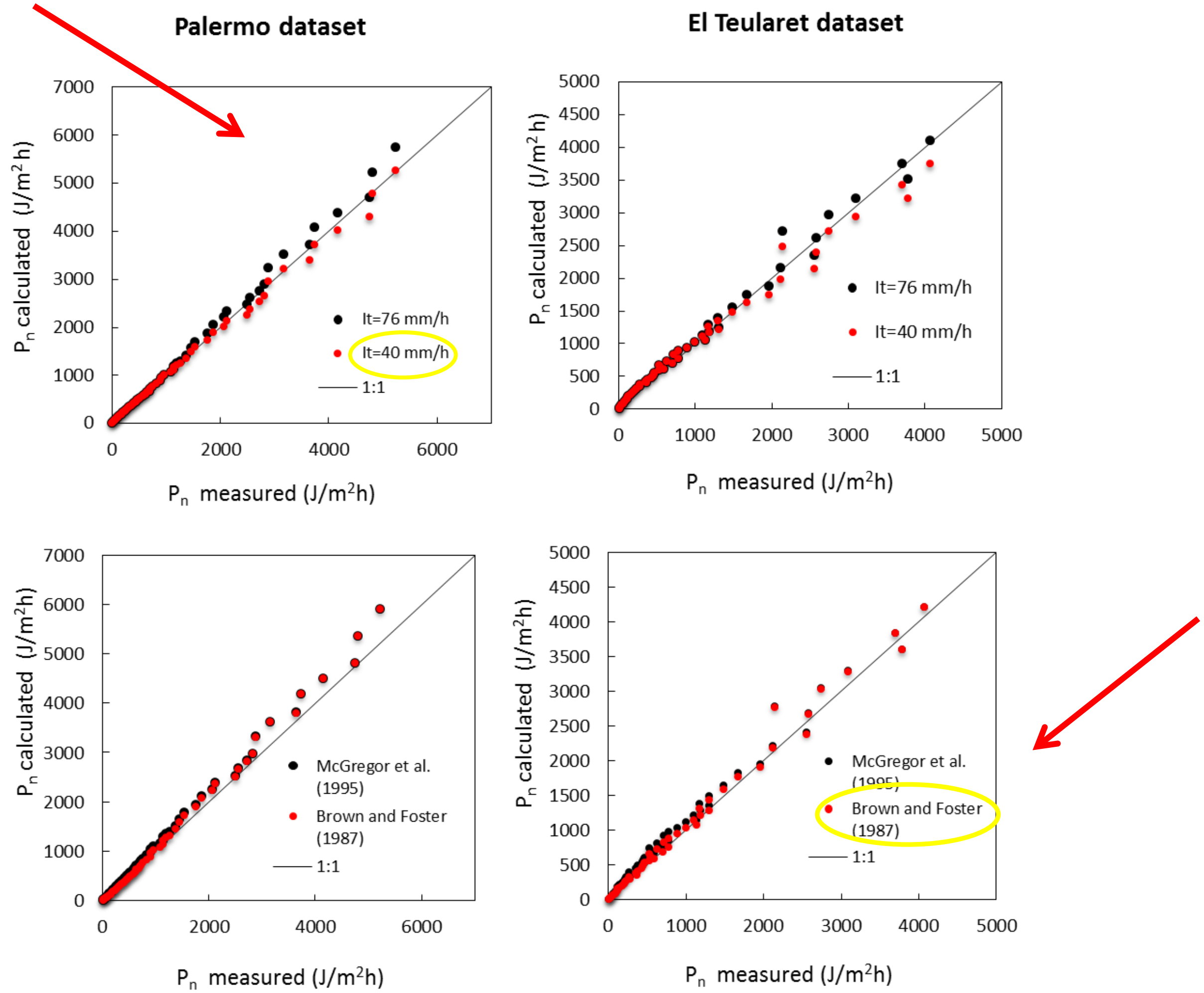
El Teularet

✓ Brown and Foster (1987):

- for *I* < 10 mm/h underestimation
- for *I* > 30 mm h^{-1} slight overestimation

✓ McGregor et al. (1995):

- for *I* < 3 mm/h slight underestimation
- for *I* > 10 mm h^{-1} systematic overestimation

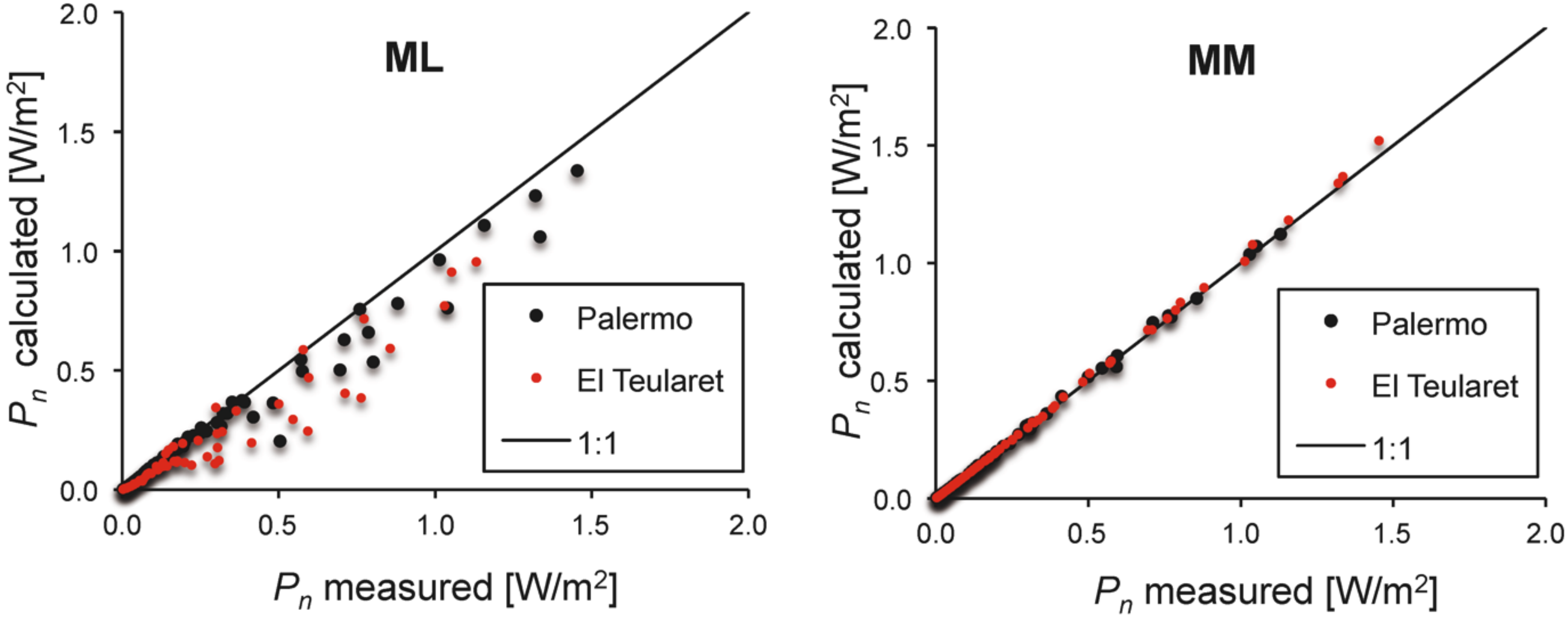


	Wischmeier and Smith (1978) It=76 mm/h		Wischmeier and Smith (1978) It=40 mm/h		Brown and Foster (1987)		Mc Gregor et al. (1995)	
	Palermo	El Teularet	Palermo	El Teularet	Palermo	El Teularet	Palermo	El Teularet
Mean error (%)	4.8	15.9	3.9	16.3	9.3	8.8	8.9	18.1
Percentage of measurements affected by an error >10% (%)	40.7	54.7	30.5	60.4	67.8	34.0	81.4	67.9

3. Results and discussions

The measured P_n values were compared with the ones obtained by

$$P_n = 10^{-6}\frac{9.5^2}{7.2}\rho\left[1-\frac{2\Lambda^{4+\mu}}{(6+\Lambda)^{4+\mu}}+\frac{\Lambda^{4+\mu}}{(12+\Lambda)^{4+\mu}}\right]I$$



- ML method yields an underestimation of P_n
- MM method yields accurate estimates of P_n

It is more important to estimate exactly D_0 than to reproduce the whole DSD

3. Results and discussions

The exponential distribution of Marshall and Palmer (1948) (when it is referred to the unit area and time) can be assumed formally identical to Ulbrich's distribution setting $\mu = 0.67$

Taking into account that

$$D_0 = \frac{3.67+\mu}{\Lambda}$$

+

from

$$\frac{P_n}{I} = 10^{-6}\frac{9.5^2}{7.2}\rho\left[1-\frac{2\Lambda^{4+\mu}}{(6+\Lambda)^{4+\mu}}+\frac{\Lambda^{4+\mu}}{(12+\Lambda)^{4+\mu}}\right]$$

$$\frac{P_n}{I} = 10^{-6}\frac{9.5^2}{7.2}\rho\left[1-\frac{2}{(6\frac{D_0}{4.34}+1)^{4.67}}+\frac{1}{(6\frac{D_0}{2.17}+1)^{4.67}}\right]$$

3. Results and discussions

$$\frac{P_n}{I} = 10^{-6}\,\frac{9.5^2}{7.2}\,\rho\left[1 - \frac{2}{(6\frac{D_0}{4.34}+1)^{4.67}} + \frac{1}{(6\frac{D_0}{2.17}+1)^{4.67}}\right]$$

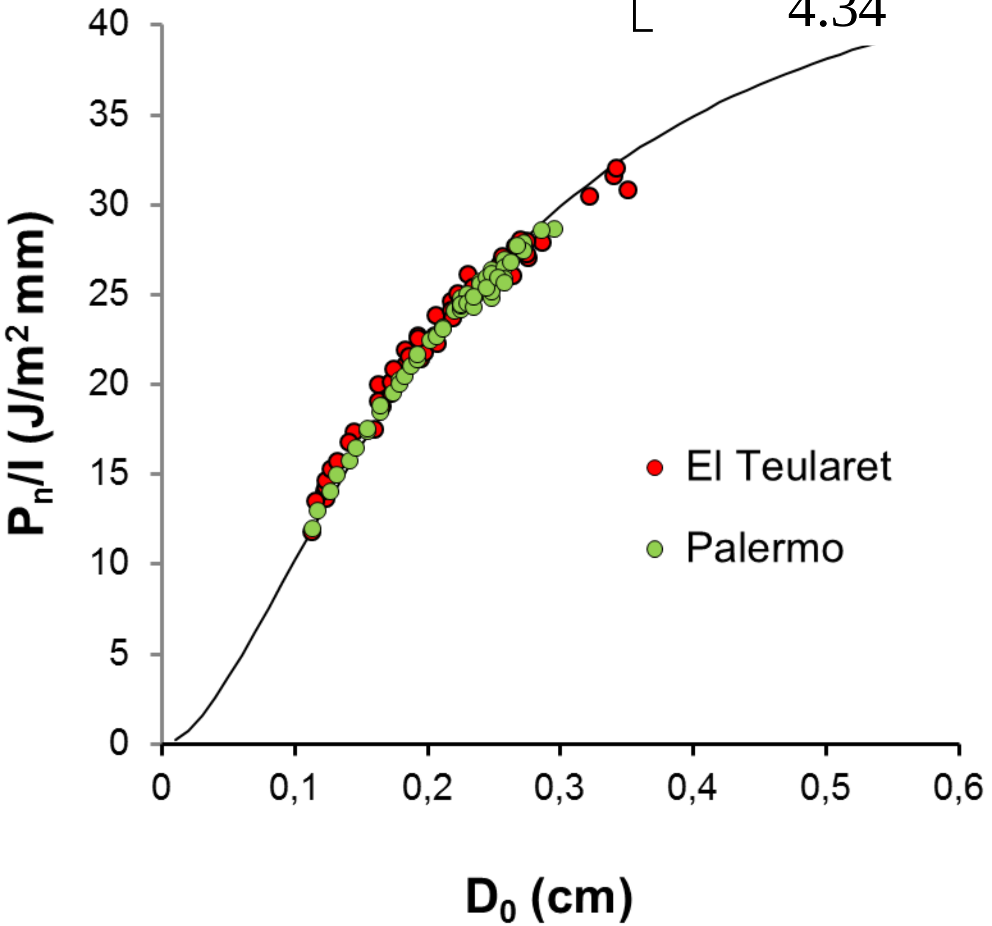
It establishes theoretically that the energy per unit volume of rainfall (P_n/I) depends only on median volume diameter

According to this relationship, if D_0 assumes a constant value P_n/I assumes a constant value too

It represents a theoretical confirmation of the Wischmeier and Smith (1978) hypothesis

3. Results and discussions

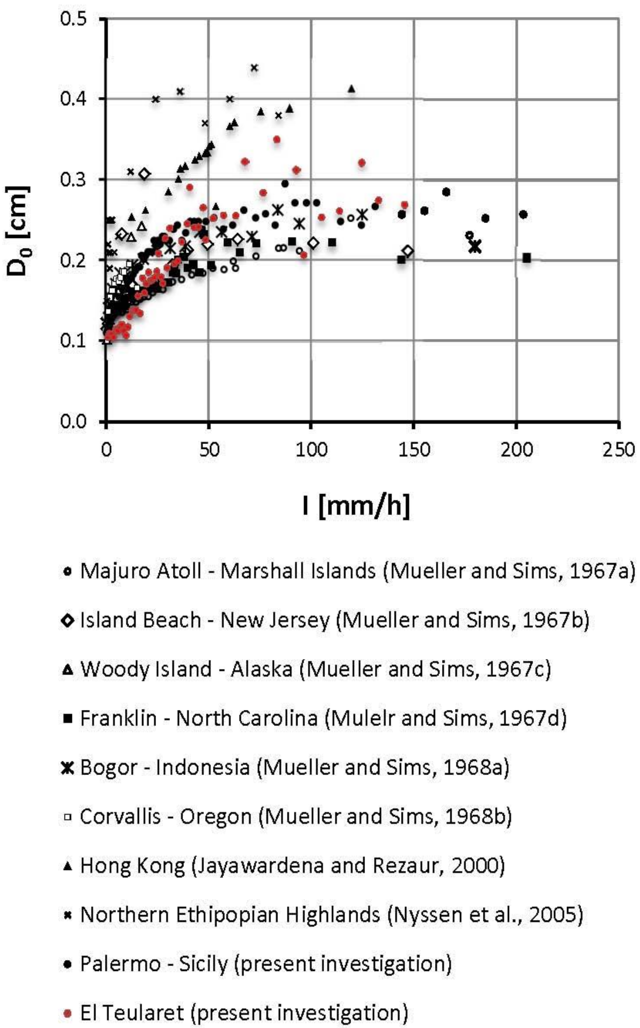
$$\frac{P_n}{I} = 10^{-6}\,\frac{9.5^2}{7.2}\,\rho\left[1 - \frac{2}{(6\frac{D_0}{4.34}+1)^{4.67}} + \frac{1}{(6\frac{D_0}{2.17}+1)^{4.67}}\right]$$

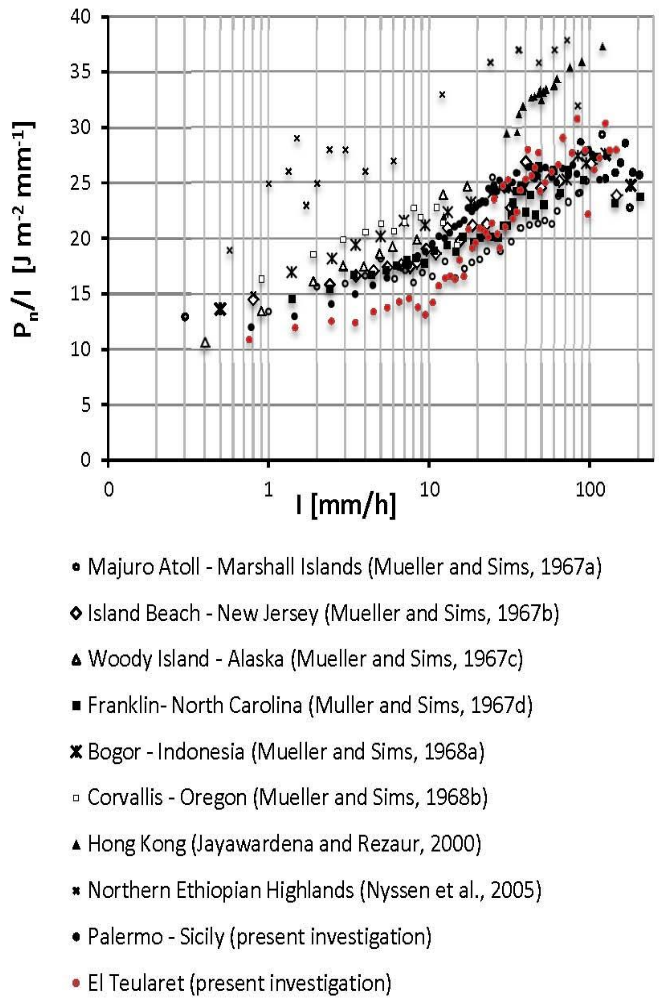


This relationship theoretically deduced (none calibration parameter was used) is very close to the experimental pairs (D_0, P_n/I) demonstrating its fully reliability for estimating kinetic power in both sites.

3. Results and discussions

Comparison with datasets obtained by different techniques in different climate conditions

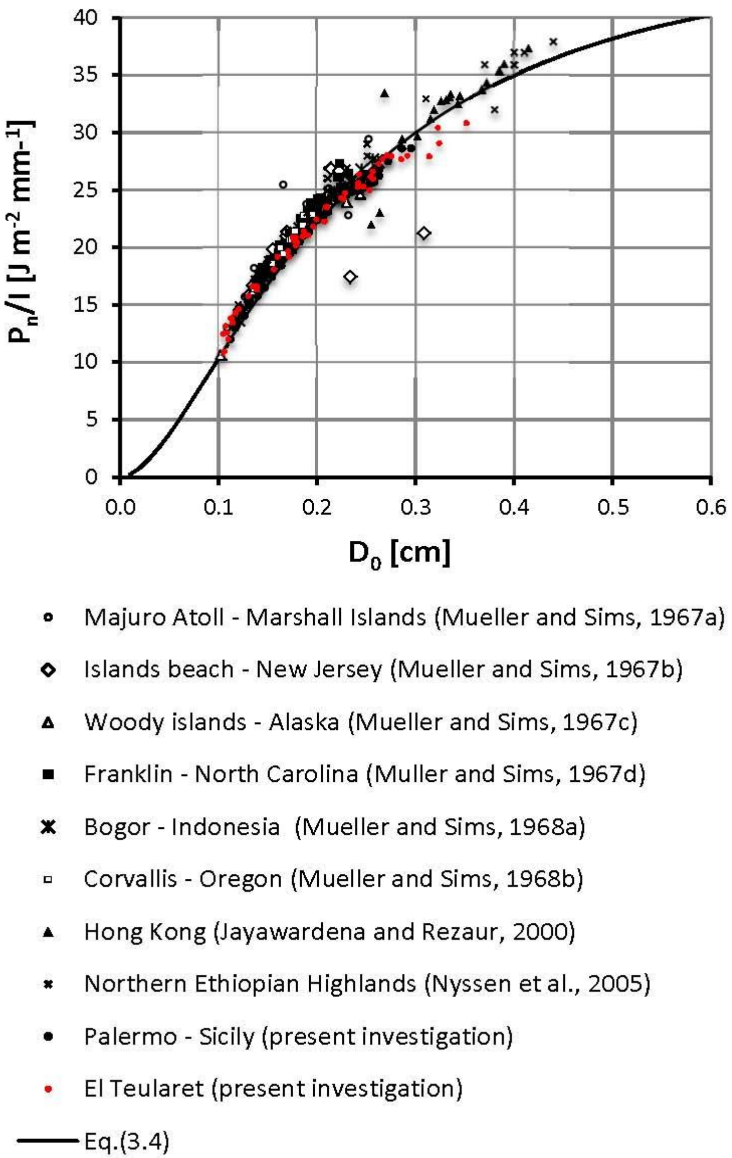




The datasets are not overlapped

Both the relationships P_n/I–I and D_0–I are site-specific and depend also on other variables (climate, rainfall type, altitude, measurement technique)

3. Results and discussions

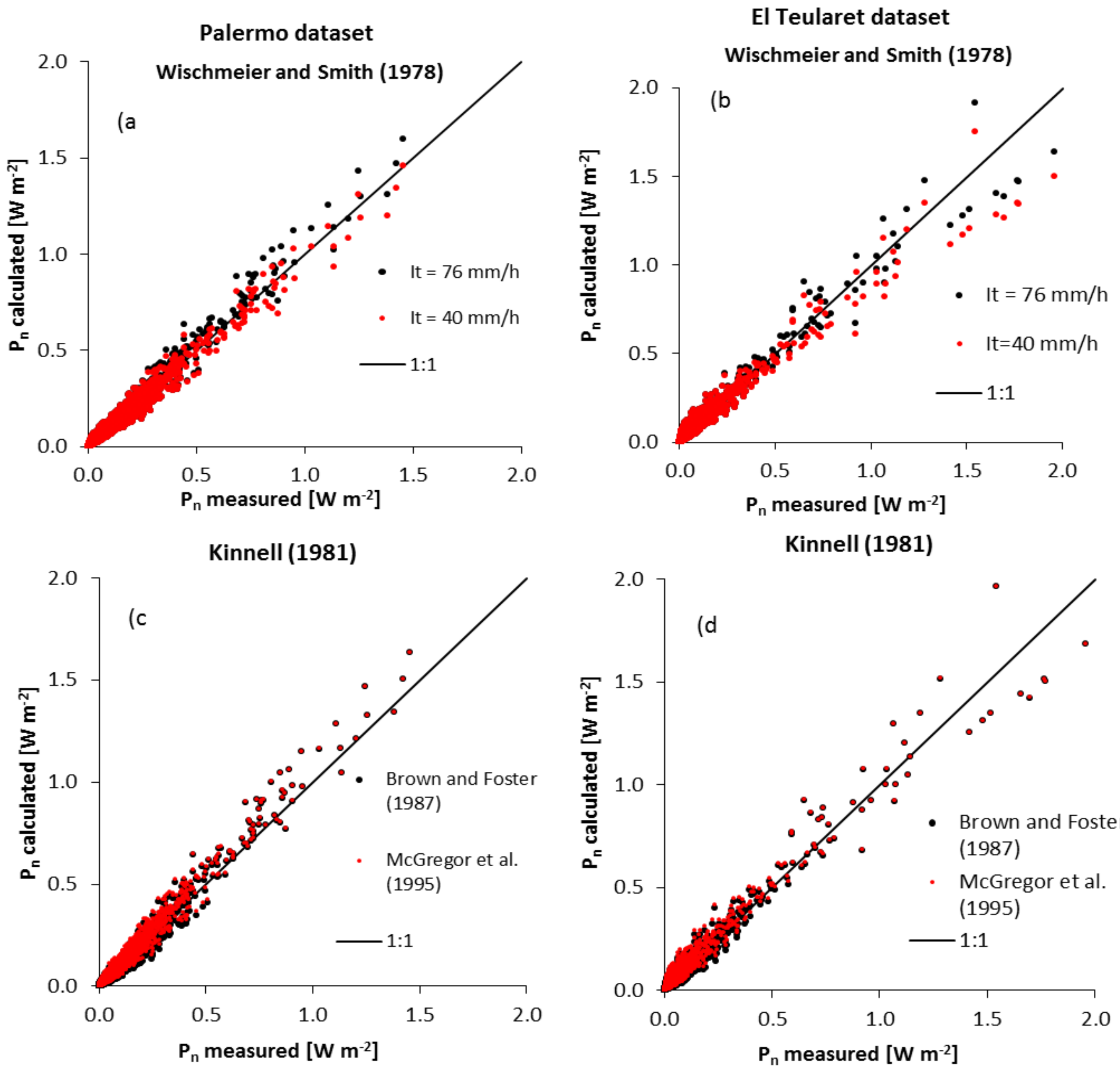


$$\frac{P_n}{I} = 10^{-6}\frac{9.5^2}{7.2}\rho\left[1 - \frac{2}{(6\frac{D_0}{4.34}+1)^{4.67}} + \frac{1}{(6\frac{D_0}{2.17}+1)^{4.67}}\right]$$

- P_n/I depends only on D_0
- the relation $P_n/I - D_0$ is not site-specific
- in contrast with Salles et al. (2002), the knowledge of both I and D_0 allows to determine the kinetic power of rainfall
- the deduced relationship is fully reliable to estimate P_n/I

3. Results and discussions

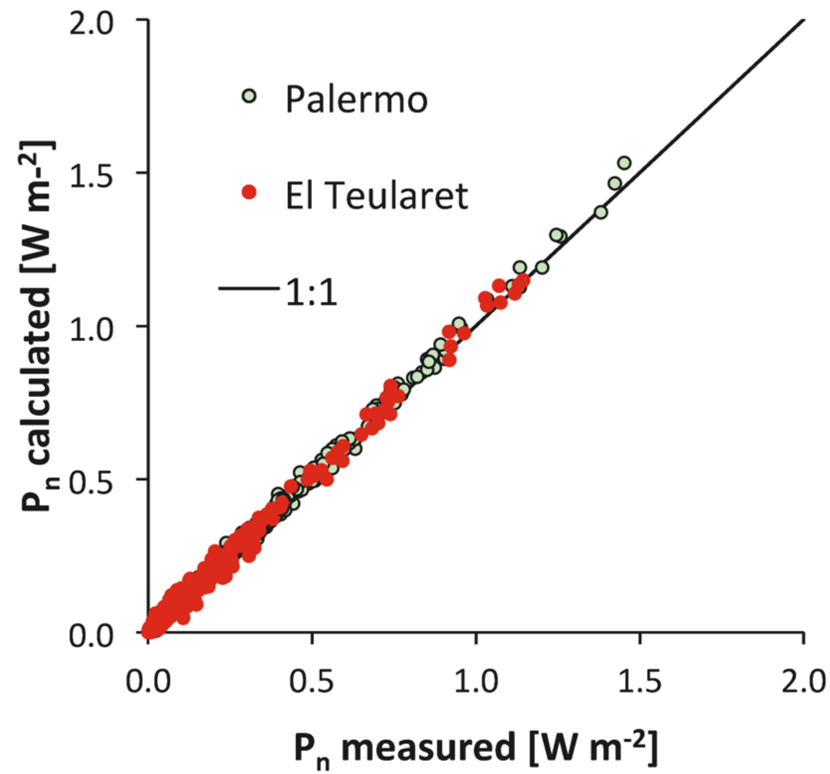
Disaggregated datasets



3. Results and discussions

Disaggregated datasets

$$\frac{P_n}{I} = 10^{-6}\frac{9.5^2}{7.2}\rho\left[1-\frac{2}{(6\frac{D_0}{4.34}+1)^{4.67}}+\frac{1}{(6\frac{D_0}{2.17}+1)^{4.67}}\right]$$



	Wischmeier and Smith (1978) It=76 mm/h		Wischmeier and Smith (1978) It=40 mm/h		Brown and Foster (1987)		Mc Gregor et al. (1995)		Eq. theoretically deduced	
	Palermo	El Teularet	Palermo	El Teularet	Palermo	El Teularet	Palermo	El Teularet	Palermo	El Teularet
Mean error (%)	25.0	58.3	24.9	58.4	23.8	40.3	22.0	48.2	5.8	7.9
Percentage of measurements affected by an error >10% (%)	73.0	81.8	73.0	81.9	76.7	82.3	74.2	81.6	14.5	27.0

4. Conclusions

- ✓ For a given rainfall intensity, the DSDs detected at Palermo result different from the El Teularet ones.
- ✓ Median volume diameter values are approximately coincident for the two datasets. D_0 increases with I until to a threshold value in agreement to Wischmeier and Smith approach.
- ✓ It was positively tested the reliability of Ulbrich's distribution to both datasets
- ✓ For both datasets, the measured kinetic power per unit volume of rainfall (P_n/I) resulted dependent only on median volume diameter.
- ✓ The Wischmeier and Smith (1978) relationship results reliable even if the threshold limit has to be set equal to 40 mm/h.
- ✓ The relationships of Brown and Foster (1987) and McGregor et al. (1995) yield an underestimation for lower values of rainfall intensity and an overestimation for the highest values of *I* and so a recalibration is needed

4. Conclusions

- ✓ A theoretical relationship was derived from the exponential distribution of Marshall and Palmer (1948) according which P_n/I depends only on median volume diameter

- ✓ This relationship results fully applicable to both datasets confirming the Wischmeier and Smith hypothesis

- ✓ The comparison of detected datasets with the ones measured in other climatic contest highlighted that rainfall intensity alone is not enough to estimate both P_n/I and D_0, confirming that the relations P_n/I-I and D_0-I are site-specific. Instead, for all available datasets, the pairs (D_0, P_n/I) result aligned around the curve representing the theoretically deduced relationship.

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