Università degli Studi di Palermo

d.m.i
matematica e informatica @ unipa

SDF
Sustainable Decision Framework

Progetto MISE n. F/310060/02/X56
CUP: B79I23000540005
COR: 14019280

Finanziato dall'Accordo per l'Innovazione del Ministero delle Imprese e del Made in Italy

BOOK OF ABSTRACTS

NEW TRENDS IN MANY-VALUED LOGIC AND CATEGORICAL ALGEBRA

An overview of the recent trends in many-valued logic and categorical algebra in the context of the SDF project scopes

wednesday, march 18th, 2026
14.00 hr
DMI, Aula 5
via Archirafi, 34
Palermo

Organising Committee
Lydia Castronovo
Giuseppe Failla
Giuseppe Filippone
Giuseppe Giacopelli
Gianmarco La Rosa
Manuel Mancini
Federica Piazza

Supported by
Progetto SDF
CUP B79J23000540005

Some properties of implicative semilattices
Corentin Vienne

On actions and split extensions in varieties of hoops
Manuel Mancini

Towards a notion of coherent and ideal actions in ideally exact contexts
Federica Piazza

A generalization of the Brauer-Fowler theorem for p-elements
Alessandro Dioguardi Burgio

Properties of continued fractions in the field of p-adic numbers
Giuliano Romeo

– Coffee Break –

Fuzzy multi-criteria decision making and conditional random quantities
Lydia Castronovo

Some Questions on Fuzziness and Nilpotency in Lie Algebras
Gianmarco La Rosa

Nonlocal Carrier's Problems
Giuseppe Failla

Polymorphic Word Representation: A mathematical framework for "Real World" LUT-driven string-pattern matchings and its applications to XAI
Giuseppe Giacopelli

Fuzzy Weighting Scheme for Artificial Experts
Giuseppe Filippone

Università degli Studi di Palermo
Dipartimento di Matematica e Informatica

NEW TRENDS IN MANY-VALUED LOGIC AND CATEGORICAL ALGEBRA

BOOK OF ABSTRACTS

Palermo
March 18th, 2026

Schedule

- 14:00 - Corentin Vienne, *Some properties of implicative semilattices*
- 14:20 - Manuel Mancini, *On actions and split extensions in varieties of hoops*
- 14:40 - Federica Piazza, *Towards a notion of coherent and ideal actions in ideally exact contexts*
- 15:00 - Alessandro Dioguardi Burgio, *A generalization of the Brauer-Fowler theorem for* p*-elements*
- 15:20 - Giuliano Romeo, *Properties of continued fractions in the field of* p*-adic numbers*
- 15:40 - **Coffe Break**
- 16:10 - Lydia Castronovo, *Fuzzy multi-criteria decision making and conditional random quantities*
- 16:30 - Gianmarco La Rosa, *Some Questions on Fuzziness and Nilpotency in Lie Algebras*
- 16:50 - Giuseppe Failla, *Nonlocal Carrier's Problems*
- 17:10 - Giuseppe Giacopelli, *Polymorphic Word Representation: A mathematical framework for "Real World" LUT-driven string-pattern matchings and its applications to XAI*
- 17:30 - Giuseppe Filippone, *Fuzzy Weighting Scheme for Artificial Experts*

Some properties of implicative semilattices

Corentin Vienne

Wallonie-Bruxelles International, Belgium.
E-mail: corentin.vienne@uclouvain.be

The goal of this talk is to present some of the results established in [1]. In that work, we investigated several aspects of the variety of implicative semilattices. By analysing commutators, normal subobjects, centralisers, and free objects in this context, we managed to understand whether or not certain categorical-algebraic conditions are satisfied by implicative semilattices, such as *algebraic coherence*, the *normality of Higgins commutators*, the *transitivity of normality*, or the *normality of unions*. If time permits, we will conclude with a few remarks on the subvariety of *equationally linear* Heyting semilattices (studied in [2]) and with some open problems.

This is joint work with Xabier García-Martínez, James Richard Andrew Gray, Michael Hoefnagel and Tim Van der Linden.

References

[1] X. García-Martínez, J. R. A. Gray, M. A. Hoefnagel, T. Van der Linden and C. Vienne, *Categorical algebraic aspects of Heyting semilattices* (2025), preprint arxiv:2508.11250.

[2] X. García-Martínez, J. R. A. Gray, M. A. Hoefnagel, T. Van der Linden and C. Vienne, *Observations on the variety of Equationally linear Heyting semilattices* (2025), in preparation.

[3] C. Vienne, *Categorical-algebraic conditions in semi-abelian categories* (2025), PhD Thesis, UCLouvain.

On actions and split extensions in varieties of hoops

Manuel Mancini

Dipartimento di Matematica e Informatica, Università degli Studi di Palermo,
via Archirafi 34, 90123, Palermo, Italy
E-mail: `manuel.mancini@unipa.it`

BL-algebras provide the algebraic semantics of *Basic Logic*, the logic of continuous t-norms, capturing the common fragment of *Łukasiewicz*, *Gödel*, and *Product* logics. It is well known that, up to isomorphism, every continuous t-norm behaves locally as one of the three fundamental ones: the Łukasiewicz t-norm $x \cdot_L y = \max\{x + y - 1, 0\}$, the Gödel t-norm $x \cdot_G y = \min\{x, y\}$, or the product t-norm $x \cdot_P y = xy$.

Each t-norm induces a residuation defined by $x \to y = \sup\{z \in [0,1] \mid z \cdot x \leqslant y\}$. The residuations associated with the three fundamental continuous t-norms were studied by P. Hájek, who provided axiomatizations of the corresponding varieties of algebras: the variety MVAlg of *MV-algebras* forms the algebraic semantics of Łukasiewicz Logic; the variety GAlg of *Gödel algebras* forms the algebraic semantics of Gödel Logic; and the variety PAlg of *product algebras* forms the algebraic semantics of Product Logic. Finally, P. Hájek introduced the variety BLAlg of *BL-algebras*, which provides the algebraic semantics of Basic Logic. From a categorical point of view, the variety BLAlg is an ideally exact category. Moreover, if $\mathbf{2}$ denotes the two-element Boolean algebra, then the semi-abelian categories $(\mathsf{BLAlg} \downarrow \mathbf{2})$, $(\mathsf{MVAlg} \downarrow \mathbf{2})$, $(\mathsf{GAlg} \downarrow \mathbf{2})$, $(\mathsf{PAlg} \downarrow \mathbf{2})$ are equivalent, respectively, to the varieties of basic, Wajsberg, Gödel, and product hoops.

The aim of this talk is to study internal actions and split extensions in the variety of hoops, with particular attention to split extensions with *strong section*. Such extensions are described in terms of *strong external actions*, i.e., a pair of maps satisfying a set of identities related to the axioms satisfied by the hoop [1, 2]. We prove that for any hoop X there is a natural isomorphism $\mathrm{EAct}_{\mathrm{ss}}(-, X) \cong \mathrm{SplExt}_{\mathrm{ss}}(-, X)$ between the functor of strong external actions on X and the functor of isomorphism classes of split extensions with strong section with kernel X. We also show that this notion trivializes for MV-algebras, while in the variety of Gödel hoops strong external actions coincide with those of basic hoops.

This is joint work with Giuseppe Metere, Federica Piazza and Marco Elio Tabacchi.

References

[1] M. Mancini, G. Metere, F. Piazza and M. E. Tabacchi, *On split extensions of product hoops*, 2025 IEEE International Conference on Fuzzy Systems (FUZZ), Reims, France, 2025, 1–5.

[2] M. Mancini, G. Metere, and F. Piazza, *On actions and split extensions in varieties of hoops: the case of strong section*, Studia Logica, to appear (2026). Preprint available at `arXiv:2510.06886`.

Towards a notion of coherent and ideal actions in ideally exact contexts

Federica Piazza

Dipartimento di Scienze Matematiche e Informatiche, Scienze Fisiche e Scienze della Terra, Università degli Studi di Messina,
viale Ferdinando Stagno d'Alcontres 31, 98166 Messina, Italy.
E-mail: federica.piazza1@studenti.unime.it

In the context of ideally exact categories, we introduce the notions of internal *coherent* action and internal *ideal* action that generalize different aspects of unital actions of rings. We prove that every ideal action is coherent, and we call BAT[†] the ideally exact contexts with a *good theory of actions*, i.e., where all coherent actions are ideal and all morphisms of such actions are ideal. Eventually, we present some case studies of BAT contexts: MV-algebras, product algebras, unital non-associative $\mathbb{F}$-algebras and rings, and $\mathsf{Set}^{\mathrm{op}}$, the dual of the category of sets.

This is joint work with Manuel Mancini and Giuseppe Metere.

References

[1] M. Mancini, G. Metere and F. Piazza, *Coherent and ideal actions in ideally exact categories* (2025), submitted, preprint arXiv:2507.06124.

[†]The acronym BAT is inspired by the notion of BIT-variety, where BIT stands for **B**uona (good, in Italian) **I**deal **T**heory, introduced by A. Ursini. Analogously, BAT stands for **B**uona **A**ction **T**heory.

A generalization of the Brauer-Fowler theorem for p-elements

Alessandro Dioguardi Burgio

Dipartimento di Matematica e Informatica, Università degli Studi di Palermo,
via Archirafi 34, 90123, Palermo, Italy
E-mail: alessandro.dioguardiburgio@unipa.it

The Brauer–Fowler theorem states that the order of a finite simple group can be bounded in terms of the orders of the centralizers of its involutions. B. Hartley, using the classification of finite simple groups, generalized in [1] the theorem to arbitrary automorphisms. In [2], S. V. Skresanov proved another version of the theorem for simple locally finite groups, replacing the order of the centralizer of an involution with the number of involutions it centralizes. In this talk, we focus on locally finite groups containing elements of prime order that centralize only finitely many elements of a given prime or prime power order. We therefore present several results in this direction.

This is joint work with Kıvanç Ersoy and Edoardo Salati.

References

[1] B. Hartley, *A general Brauer-Fowler theorem and centralizers in locally finite groups*, Pacific Journal of Mathematics **152** (1992), no. 1, 101–117.

[2] S. V. Skresanov, *A generalization of the Brauer–Fowler theorem*, Journal of Group Theory **27** (2024), no. 6, 1197–1202.

Properties of continued fractions in the field of p-adic numbers

Giuliano Romeo

Dipartimento di Matematica, Università di Trento, Via Sommarive 14, 38123, Povo (TN), Italy
E-mail: giuliano.romeo@unitn.it

Continued fractions are a central tool in Diophantine approximation, the branch of number theory that investigates the quality of approximations of irrational numbers by means of rationals. Because of their optimal properties over real numbers, continued fractions have been studied also over the field of p-adic numbers $\mathbb{Q}_p$.

In the first part of this talk, we provide an introduction on the history and the main features of continued fractions and p-adic numbers. Then, we put together these elements and we define some algorithms for the construction of p-adic continued fractions. In the last part of the talk, we present some selected results from [1, 2, 3, 4].

The talk is based on joint works with Nadir Murru and Giulia Salvatori.

References

[1] N. Murru, G. Romeo, *A new algorithm for p–adic continued fractions*, Mathematics of Computation **93** (2024), no. 347, 1309–1331.

[2] G. Romeo, *Continued fractions in the field of p–adic numbers*, Bulletin of the American Mathematical Society **61** (2024), no. 2, 343–371.

[3] G. Romeo, *Real convergence and periodicity of p–adic continued fractions*, The Ramanujan Journal **68** (2025), no. 112.

[4] G. Romeo, G. Salvatori, *The arithmetic of continued fractions in the field of p-adic numbers* (2026), preprint arXiv:2512.11069.

Fuzzy multi-criteria decision making and conditional random quantities

Lydia Castronovo

Dipartimento di Matematica e Informatica, Università degli Studi di Palermo,
via Archirafi 34, 90123, Palermo, Italy
E-mail: lydia.castronovo@unipa.it

In the framework of (fuzzy) multi-criteria decision making [1, 2], we propose a method that allows the decision maker to subjectively approach the problem by suitably modifying the decision matrix. Following the recent approach proposed in [3], we use the conditional probability interpretation of membership functions and the operations among conditionals in the framework of conditional random quantities to model logical and probabilistic operations among the columns of the decision matrix seen as particular fuzzy sets. We consider a decision problem related to a random quantity X with set of values $\{x_1, x_2, \ldots, x_n\}$. A meta-expert $\mathcal{E}$ chooses the relevant properties $\{C_1, C_2, \ldots, C_m\}$ of X. In this setting, the properties C_j are the criteria of the decision problem and the alternatives are represented by the events $A_i = (X = x_i)$ for $i = 1, \ldots, n$. To build the decision matrix, the decision maker has to set the criteria's weights w_j and the scores a_{ij}, interpreted as the probabilities of particular events and conditional events, respectively. Then, in this setting we allow logical operations among criteria by exploiting the conditional probability interpretation. More precisely, when considering the complement, conjunction and disjunction of criteria, we build the complement, intersection and union of the corresponding fuzzy sets. The conditional probability interpretation of the scores helps us find the new scores of the modified decision matrix, which retains all the original criteria, as well as the new columns given by the logical operations considered by the decision maker.

This talk is based on a joint work with Giuseppe Filippone, Gianmarco La Rosa, Giuseppe Sanfilippo and Marco Elio Tabacchi [2].

References

[1] L. Castronovo, G. Filippone, M. Galici, G. La Rosa and M. .E. Tabacchi, *Fuzzy MCGDM Approach for Ontology Fuzzification*, Electronics **14** (2025), 3596.

[2] L. Castronovo, G. Filippone, G. La Rosa; G. Sanfilippo and M. .E. Tabacchi, *Logic and Probabilistic Operations on the Decision Matrix in a Fuzzy Multi-Criteria Decision-Making Problem*, Mathematics **14** (2026), 778.

[3] L. Castronovo and G. Sanfilippo, *Compound Conditionals and Fuzzy Sets*. In: J. Ansari et al. Combining, Modelling and Analyzing Imprecision, Randomness and Dependence. SMPS 2024. Advances in Intelligent Systems and Computing, vol 1458. Springer, Cham.

Some Questions on Fuzziness and Nilpotency in Lie Algebras

Gianmarco La Rosa

Dipartimento di Matematica e Informatica, Università degli Studi di Palermo,
via Archirafi 34, 90123, Palermo, Italy
E-mail: gianmarco.larosa@gmail.com

This talk addresses structural questions in the theory of fuzzy Lie algebras, viewed as an extension of classical Lie algebra theory within the framework of fuzzy set theory. Treating fuzzy Lie algebras as fuzzy vector spaces endowed with a compatible bracket structure, we investigate how fundamental algebraic notions adapt to the fuzzy setting. Particular attention is devoted to the interplay between structural properties and fuzziness. We discuss conditions under which classical linear-algebraic concepts admit coherent fuzzy counterparts and present representative results clarifying their behavior in fuzzy Lie subalgebras.

Within this structural perspective, a new definition of nilpotency for fuzzy Lie algebras is introduced and compared with both the classical notion and existing alternatives. This comparison sheds light on how nilpotent phenomena can be meaningfully characterized in the fuzzy framework. Overall, the results contribute to a more systematic understanding of fuzzy Lie algebras and aim to support further theoretical and computational developments in fuzzy algebraic structures.

This talk is based on joint work with Giuseppe Filippone, Mario Galici, Federica Piazza and Marco Elio Tabacchi.

References

[1] G. Filippone, M. Galici, G. La Rosa, F. Piazza and M. E. Tabacchi, *A new definition of fuzzy nilpotent Lie algebras*, 2025 IEEE International Conference on Fuzzy Systems (FUZZ), Reims, France, 2025, 1–5.

[2] G. Filippone, M. Galici, G. La Rosa and M. E. Tabacchi, *Some Results on Fuzzy Basis of Fuzzy Lie Algebras*, Advances in Fuzzy Logic and Technology, LNCS 15884, 2025 Springer.

Nonlocal Carrier's Problems

Giuseppe Failla

Dipartimento di Matematica e Informatica, Università degli Studi di Palermo,
via Archirafi 34, 90123, Palermo, Italy
E-mail: giuseppe.failla02@unipa.it

We investigate the existence and multiplicity of positive solutions for the following elliptic nonlocal nonvariational problem

$$\begin{cases} -a\left(\int_\Omega u^q \mathrm{d}x\right)\Delta_p u = f(x,u) \quad \text{in} \quad \Omega; \\ u > 0 \quad \text{in} \quad \Omega; \\ u = 0 \quad \text{on} \quad \partial\Omega; \end{cases} \tag{P}$$

where $\Omega \subset \mathbb{R}^N$ be a bounded domain with smooth boundary, $f\colon \Omega \times \mathbb{R} \to \mathbb{R}$ be a Carathéodory function satisfying suitable structural assumptions and $a\colon [0,+\infty[\to \mathbb{R}$ be a continuous sign-changing function depends on the L^q-norm of the solution. The lack of a variational structure, prevent the direct application of standard theory. To overcome these difficulties, we develop an approach based on a combination of variational arguments, truncation techniques, sub- and super-solutions methods, and tools from set-valued analysis. A suitable one-dimensional fixed-point map associated with the nonlocal term plays a crucial role in deriving the existence of pairs of positive solutions. Moreover, we provide three nonexistence results under additional monotonicity conditions on f.

References

[1] P. Candito, G. Failla, L. Gasiński and R. Livrea, *Multiple positive solutions for a quasilinear nonlocal problema via topological, variational and set-valued methods*, sumbitted.

[2] G. Failla, L. Gasiński and J. R. Santos Júnior, *Nonexistence results and multiple solutions for p-Laplacian nonlocal problem*, submitted.

Polymorphic Word Representation: A mathematical framework for "Real World" LUT-driven string-pattern matchings and its applications to XAI

Giuseppe Giacopelli

Dipartimento di Matematica e Informatica, Università degli Studi di Palermo,
via Archirafi 34, 90123, Palermo, Italy
E-mail: giuseppe.giacopelli@unipa.it

LLMs and AI are changing the world we live in. A user can chat with a chatbot, make it browse the web to collect additional information, or make it interact with the real world to fulfill a particular task. However, as the AI Act points out [1], critical applications require systems with high standards of security, reliability, and explainability. This need for smarter, deterministic, and inspectable systems has increased interest in the concept of XAI (Explainable Artificial Intelligence) [2]. In this section, an alternative approach to *LLM tool call* based on *Polymorphic Word Representation* is described. The underlying idea is to assign to each word in a query a set of attributes based on simple deterministic heuristics. The objective is to solve the problem associated with assigning a single best attribute to each word in order to maximize the fit of the query to a string-pattern from a given pattern set (linked to the AI tools). A simple deterministic approximate solution for the problem will be found, and it will be pointed out how this problem can be related to discrete line coloring optimal for a given set of constraints [4]. In conclusion, some *real world* examples and considerations about LLM best practices to reduce or mitigate the LLM hallucination problem will also be presented [3].

References

[1] AI Act Info, *AI Act Info*. https://aiactinfo.eu/

[2] L. Longo, M. Brcic, F. Cabitza, J. Choi, R. Confalonieri, J. Del Ser, R. Guidotti, Y. Hayashi, F. Herrera, A. Holzinger, R. Jiang, H. Khosravi, F. Lecue, G. Malgieri, A. Páez, W. Samek, J. Schneider, T. Speith, S. Stumpf, "Explainable Artificial Intelligence (XAI) 2.0: A manifesto of open challenges and interdisciplinary research directions", *Information Fusion* **106** (2024), 102301.

[3] L. Huang, W. Yu, W. Ma, W. Zhong, Z. Feng, H. Wang, Q. Chen, W. Peng, X. Feng, B. Qin and T. Liu, *A Survey on Hallucination in Large Language Models: Principles, Taxonomy, Challenges, and Open Questions*, ACM Transactions on Information Systems **43** (2025), no. 2.

[4] D. Y. Kang, T. Kelly, D. Kühn, A. Methuku and D. Osthus, *Graph and hypergraph colouring via nibble methods: A survey* (2023), European Congress of Mathematics.

Fuzzy Weighting Scheme for Artificial Experts

Giuseppe Filippone

Dipartimento di Matematica e Informatica, Università degli Studi di Palermo,
via Archirafi 34, 90123, Palermo, Italy
E-mail: giuseppe.filippone01@unipa.it

In many applications, the decision-making process involves multiple users. It may be necessary to assign weights to them in order to calculate the best solution. There is no standard for assigning weights, as it depends on which users are involved and the decision problem. Furthermore, it can be particularly difficult to assign weights in the case of multiple evaluation criteria or when expert users are involved. Selecting the appropriate weight for each human expert may raise a considerable challenge.

In our approach, human experts are replaced by artificial ones (e.g., algorithms), the weight of which can be computed using more deterministic methods. In particular, based on the decision problem, the most suitable algorithms related to the problem are selected as experts. We determine the weight of each algorithm (expert) by modelling a *Fuzzy Rule-Based System* (FRBS) [3, 5]. FRBSs express their knowledge base using IF-THEN rules whose antecedents and consequents are fuzzy statements. This framework provides a structured approach for defining a straightforward, deterministic method for weighting a pool of experts in a given field. This approach has the potential to be utilised in a variety of applications involving a group of experts, such as *Multi-Criteria Group Decision-Making* MCGDM. In future, we plan to use the proposed framework to weigh experts involved in the fuzzyification of an ontology using a MCGDM framework [1, 2, 4].

References

[1] L. Castronovo, G. Filippone, M. Galici, G. La Rosa, and M. E. Tabacchi. Fuzzy MCGDM Approach for Ontology Fuzzification. *Electronics*, 14(18), 2025.

[2] L. Castronovo, G. Filippone, M. Galici, G. La Rosa, and M. E. Tabacchi. Ontology Aggregation with Maximum Consensus Based on a Fuzzy Multi-criteria Group Decision-Making Method. In Michał Baczyński, Bernard De Baets, Michal Holčapek, Vladik Kreinovich, and Jesús Medina, editors, *Advances in Fuzzy Logic and Technology*, pages 76–87, Cham, 2025. Springer Nature Switzerland.

[3] Oscar Cordón, Francisco Herrera, Frank Hoffmann, and Luis Magdalena. *Fuzzy Rule-Based Systems*, volume 19, chapter 1, pages 1–46. WORLD SCIENTIFIC, 2001.

[4] G. Filippone, G. La Rosa, and M. E. Tabacchi. SDF-FuzzIA: A Fuzzy-Ontology Based Plug-in for the Intelligent Analysis of Geo-Thematic Data. In Sébastien Destercke, Maria Vanina Martinez, and Giuseppe Sanfilippo, editors, *Scalable Uncertainty Management*, pages 163–169, Cham, 2025. Springer Nature Switzerland.

[5] J. M. Mendel. *Explainable Uncertain Rule-Based Fuzzy Systems*. Springer Cham, 3 edition, 2024.