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MAXIMUM ROLL ANGLE ESTIMATION OF A SHIP IN CONFUSED SEA WAVES VIA A QUASI-DETERMINISTIC APPROACH

G. Malara[*], P.D. Spanos[†], F. Arena[*]

[*] Department of Mechanics and Materials (MecMat)
Mediterranea University of Reggio Calabria
Loc. Feo di Vito, 89100 Reggio Calabria, Italy
e-mail: giovanni.malara@unirc.it – arena@unirc.it

[†] Department of Civil and Environmental Engineering
Rice University
MS 318, 77005 Houston, Texas, USA
e-mail: spanos@rice.edu

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Parole chiave: Rollio, sistema non-lineare, teoria di Quasi-Determinismo, simulazioni Monte Carlo, bacino di sicurezza.

Abstract. *This paper considers the maximum roll angle of a ship in confused sea waves. The roll motion is described by a nonlinear equation of motion including quadratic damping and cubic stiffness terms. The wave load is represented by a Gaussian random process of a given power spectral density. It is shown that a reliable estimate of the maximum roll angle can be determined by the (approximate) quasi-deterministic representation of the load process. This formulation provides a spectrum compatible representation of the realizations of extreme loads in the time domain. Further, the paper discusses the stability of the ship in the space of the initial conditions and outlines the advantages of the proposed approximation.*

Sommario. *La memoria analizza il massimo angolo di rollio di una nave in onde aleatorie. Il moto di rollio è studiato mediante un'equazione del moto non-lineare che include smorzamento quadratico e rigidezza cubica. Il carico è rappresentato tramite un processo aleatorio Gaussiano di data densità di potenza. Si dimostra che una stima affidabile dell'angolo massimo di rollio si può ottenere tramite una rappresentazione (approssimazione) quasi-deterministica del carico. Tale formulazione fornisce una rappresentazione spettro-compatibile delle realizzazioni di carichi estremi nel dominio del tempo. Inoltre, la memoria analizza la stabilità della nave nello spazio delle condizioni iniziali e delinea i vantaggi dell'approssimazione proposta.*

1 INTRODUCTION

A safety and reliability based design of ships relies on several approximate techniques for system response determination in random waves. These techniques involve the modelling of a 6DOF system subject to random loads. In ship dynamics terminology, these degrees of freedom are commonly referred to as heave, sway, surge, yaw, pitch and roll. One of the fundamental requirements of ship design is ensuring stability against capsizing. In this regard, the most critical degree of freedom is roll [1].

Roll estimation can be pursued by simplified models. Specifically, the roll motion is modelled as a SDOF oscillator, where nonlinearities are included. Nonlinearities are due to the restoring moment, utilized for keeping the ship in stable position; and to the viscous damping, that relates to the fluid behaviour on the ship boundary. More precisely, the former relates to the shape of the righting arm diagram. This diagram shows that the restoring moment is a nonlinear function of the roll angle. Thus, it is often approximated by a polynomial of odd powers, which yields nonlinear stiffness terms [2]. The latter relates to viscosity and to the generation of vortexes. Damping is not derived by analytical arguments. Instead, it is artificially added to the equation of motion. Two models are commonly utilized: a drag-type model, which includes quadratic damping, and a cubic model [3]. Accounting for realistic situations suggests using a stochastic model for the excitation. In this regard, the most common model, the sea state theory [4], represents the free surface displacement by a stationary normal process, so that, in linear water waves, hydrodynamic excitations are stationary normal processes, as well.

Monte Carlo simulation is, currently, the most versatile method for determining system response statistics [5], as nonlinearities are easily accounted for. The common procedure for calculating response statistics is to synthesize an adequately long time history of the system's input which is compatible with a specified power spectral density. Then, the response is determined by numerical integration of the equation of motion. In the context of ship dynamics, detailed analysis of extreme response in the time domain is required as well [6], because severe operational conditions may become critical for the ship performance. Nonetheless, Monte Carlo simulations exhibit a remarkable drawback: computational cost. Indeed, they are time consuming even with the advent of modern computational power. This disadvantage is due to the need of quantifying events with a quite low probability of occurrence.

This paper presents an approach for estimating the maximum roll angle of a ship. The ship is modelled as a nonlinear oscillator subject to stationary mean-zero normal load of a given power spectral density. The approximation relies on a quasi-determinism based representation of the wave load [7-8]. Such a representation approximates the normal load by the normalized autocovariance function of the load process, in the presence of a large maximum. The concept has been adopted for several years in the context of probabilistic wave mechanics. Herein, this formalism is used as a tool for synthesizing time variations of a large excitation which are spectrum compatible. The usefulness and reliability of the method is assessed by comparison with results of Monte Carlo simulations.

Next, the ship stability is investigated by the safe basin in the space of initial conditions [9]. It is shown that integrity curves can be constructed to identify the critical wave excitation amplitude associated with a certain degree of "erosion" of the safe basin. In this context, the advantage of the quasi-deterministic formulation relates to the possibility of generating quasi-impulsive time variations of the excitation which are spectrum compatible.

2 THEORETICAL BACKGROUND

2.1 Mathematical treatment of the roll motion

The ship roll motion equation is formulated following an approximation utilized in several studies [10]. Specifically, the motion is assumed to be governed, approximately, by the nonlinear equation

$$\ddot{\phi} + F(\dot{\phi}) + G(\phi) = M(t), \tag{1}$$

in which ϕ denotes the roll angle, $F(\dot{\phi})$ denotes nonlinear damping, $G(\phi)$ denotes nonlinear stiffness and $M(t)$ is the wave exciting moment. The damping term relates to viscous and pressure drags generated by the relative velocity between flow field and structure. In the following, a linear-plus-quadratic model is utilized. The stiffness term $G(\phi)$ relates to the restoring moment of the ship. Specifically, it is related to the righting arm dependence on roll angle. In the following, the righting arm dependence is approximated by a linear-plus-cubic term. This approximation is reliable only for moderate values of the roll angle (<35° [11]), but it is often utilized since ships are not expected to experience larger angles of motion.

Thus, the equation of roll motion (1) is recast in the form

$$\ddot{\phi} + a_1\dot{\phi} + a_2\dot{\phi}|\dot{\phi}| + a_3\phi + a_4\phi^3 = M(t), \tag{2}$$

where the a_i-coefficients (i = 1,2,3,4) depend on ship characteristics and can be estimated by system identification procedures [10]. The wave exciting moment $M(t)$ is modelled as stationary normal process of a given power spectral density. This assumption is consistent with the classical representation of random free surface displacement in linear water waves [4]. Indeed, the power spectral density of $M(t)$ is commonly determined from the free surface displacement spectrum through calculation of a transfer function pertaining to the specific ship under examination. In this regard, it is known that the spectrum of $M(t)$ can be approximated quite satisfactorily by filtered white noise [12], as long as the excitation is a stationary process. The approximating dimensionless spectrum is given by

$$\hat{S}_M(w) = Gw^4[(w^2 - k_1)^2 + (c_1 w)^2]^{-1}[(w^2 - k_2)^2 + (c_2 w)^2]^{-1}, \tag{3}$$

where G, k_1, k_2, c_1, c_2 are parameters determined by fitting the target spectrum via least-square algorithm, and w is a dimensionless frequency ($\equiv \omega T_n/2\pi$; ω denoting radian frequency and T_n undamped natural frequency). Eq. (3) can approximate a variety of spectral shapes, which include both single peaked to double peaked spectra.

2.2 Approximation of large excitations in the time domain via the quasi-deterministic approach

Assume that the wave moment $M(t)$ has a given value $M(t_0)$ at a certain time instant $t = t_0$, and derive the conditional statistics. The conditional mean $\overline{M}(t_0 + \tau)$ and the conditional standard deviation $\sigma_{M(t)|M(t_0)}(t_0 + \tau)$ of the excitation are readily determined by relying on the normality of the process $M(t)$. Specifically, after some algebraic manipulations, it can be proven that

$$\overline{M}(t_0 + \tau) = M(t_0)R(\tau)/R(0), \tag{4}$$

and

$$\sigma_{M(t)|M(t_0)}(t_0 + \tau) = \sigma_M \sqrt{1 - R^2(\tau)/R^2(0)} < \sigma_M, \tag{5}$$

where τ is the time variable; $R(\tau)$ ($\equiv E[M(t)M(t+\tau)]$; with $E[\cdot]$ denoting the mathematical expectation operator) is the autocovariance function of the excitation; and σ_M is the standard deviation of the wave moment excitation $M(t)$.

The crucial point of the quasi-determinism based formulation is the calculation of the ratio of the conditional standard deviation (5) over the conditional mean (4),

$$\sigma_{M(t)|M(t_0)}(t_0+\tau)/\overline{M}(t_0+\tau)=[\sigma_M/M(t_0)]\cdot\sqrt{R^2(0)/R^2(\tau)-1}. \quad (6)$$

This ratio is investigated in the asymptotic condition $M(t_0)/\sigma_M \to \infty$. That is, the wave exciting moment $M(t_0)$ is supposed extremely large with respect to the standard deviation of the wave moment. In this context, it is seen that

$$\sigma_{M(t)|M(t_0)}(t_0+\tau)/\overline{M}(t_0+\tau)\to 0. \quad (7)$$

Thus, the conditional standard deviation $\sigma_{M(t)|M(t_0)}(t_0+\tau)$ is quite small compared to the conditional mean $\overline{M}(t_0+\tau)$ in the vicinity of the extreme excitation. Consequently, the realizations of the process $M(t)$ can be approximated by the deterministic function (4) in the vicinity of t_0, as $M(t_0)/\sigma_M \to \infty$.

Such an approximation is exact in the asymptotic condition $M(t_0)/\sigma_M \to \infty$. Nevertheless, it renders a good approximation even for finite values of $M(t_0)$. Indeed, Boccotti [7] showed that the approximation is reliable as $M(t_0)/\sigma_M \geq 3$, by comparing theoretical results with Monte Carlo simulations of normal processes of a given power spectral density.

The properties of the quasi-deterministic excitation are defined by the normalized autocovariance function on the right hand side of eq. (4). Thus, $\overline{M}(t_0+\tau)$ is symmetrical with respect to $t = t_0$ and, at this time instant, has an absolute maximum. This maximum is exactly equal to $M(t_0)$.

Next, the quasi-deterministic excitation is specified using the power spectral density (3). Indeed, the Wiener-Khinchin theorem relates the autocovariance function to the power spectrum of the excitation. Thus,

$$\overline{M}(t_0+\tau)=M(t_0)\cdot\int_0^\infty \hat{S}_M(w)\cos(2\pi w\tau/T_n)dw \Big/ \int_0^\infty \hat{S}_M(w)dw \quad (8)$$

Fig. 3 shows the quasi-deterministic excitation in the time domain. The calculation pertains to the case of a single peaked frequency spectrum, which is typical in the context of wind-generated waves. It has been assumed $t_0 = 0$. The figure shows that the excitation is quasi-impulsive. Further, note that the approximation is not limited only to the time instants after the occurrence of the extreme excitation, but is valid in the vicinity of t_0.

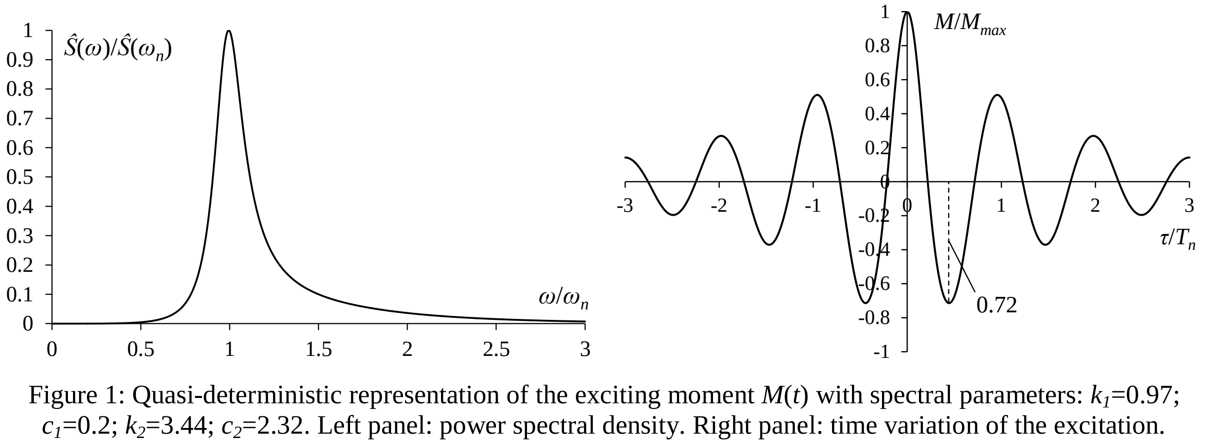

Figure 1: Quasi-deterministic representation of the exciting moment $M(t)$ with spectral parameters: k_1=0.97; c_1=0.2; k_2=3.44; c_2=2.32. Left panel: power spectral density. Right panel: time variation of the excitation.

3 RELIABILITY OF THE QUASI-DETERMINISTIC APPROXIMATION BY COMPARISON TO MONTE CARLO SIMULATIONS

The reliability of the quasi-determinism based formulation is next assessed against the results of Monte Carlo simulations. Herein, the simulations are utilized for determining the standard deviation of the roll angle process from realizations of spectrum compatible excitations. The reliability of the approximation is investigated by calculating the ratio

$$\zeta = \bar{\phi}_{\max} / \sigma_{\phi} , \tag{9}$$

with $\bar{\phi}_{\max}$ being the maximum roll angle obtained by the quasi-deterministic approximation and σ_{ϕ} the standard deviation of the roll angle obtained by the Monte Carlo simulation. The calculation involves several ship characteristics (a_i-parameters; i=1,2,3,4) and various spectral shapes of the excitation.

The maximum roll angle and the standard deviation σ_{ϕ} are obtained by numerical integration of eq. (2) via a fourth order Runge-Kutta algorithm [13], where quiescent initial conditions are assumed. The maximum roll angle is calculated by exposing the system to the excitation (8), where $M(t_0) = 3\sigma_M$. The integration is executed over a time window of $20T_n$, and the time instant $t_0 = 10T_n$, so that the maximum roll angle is independent of the transient motion influenced by the initial conditions. The standard deviation σ_{ϕ} is calculated from the time history of the response, which is determined starting from 1000 cycles of a spectrum compatible excitation, generated via FFT algorithm [5].

Two sets of numerical simulations and quasi-deterministic responses are produced. A first set involves various systems excited by a certain excitation. A second one involves a certain system exposed to various (spectrum-wise) excitations. In the former case, the a_i-coefficients ranges in the intervals a_1=[0.06, 0.15], a_2=[0, 0.8], a_3=[0.5, 1] and a_4=[0, 3], as obtained by Roberts, Dunne et al. [10] and Vasta and Roberts [11]. The spectrum of the excitation is the one shown in fig. 1, in which G = 0.03. In the latter case, the system parameters are a_1=0.012, a_2=0.35, a_3=0.504, a_4=3, which relates to a system highly nonlinear both in damping and in stiffness. The spectral parameters range in the interval G=[0.043,0.051], k_1=[0.84,0.97], c_1=[0.2,0.4], k_2=[2.15,3.44], c_2=[0.5,2.32], so that the spectral shapes vary from single peaked to double peaked spectra. Thus, the responses are representative of various wave conditions.

Fig. 2 shows the coefficient (9) calculated for both set of data. It is seen that the coefficient is always larger than 1. The mean value of ζ is 2.2, and the standard deviation is 0.15. Thus, from a holistic perspective, it is seen that the proposed approximation renders an estimate which is conservative and consistent with results of Monte Carlo simulations.

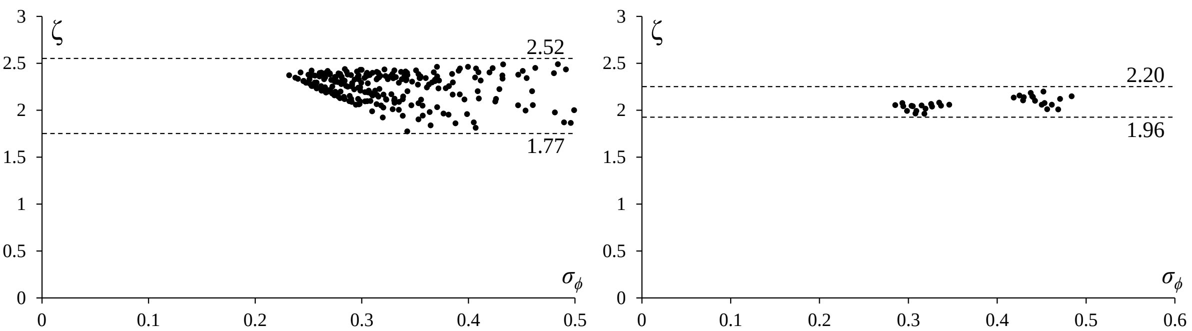

Figure 2: Coefficient (9) calculated by assuming several system parameters under a certain excitation (left panel) and a certain system subject to various excitations (right panel).

4 STABILITY CRITERION UTILIZING QUASI-IMPULSIVE EXCITATION AS WORST-CASE SCENARIO

4.1 Safe basin in the space of initial conditions

The quasi-determinism approximation is used in conjunction with the safe basin in the space of initial conditions [9]. The safe basin provides a global view on the performance of a ship in extreme waves. This approach is quite effective for the ship stability assessment, as it includes the effect of the ship initial conditions at the beginning of an extreme event. Thus, it is more rational than any "conventional" steady state analysis. In this context, the proposed approximation provides a worst-case scenario, which is physically consistent, being spectrum compatible, and quasi-impulsive, as expected in extreme conditions. In this context, the quasi-determinism based formulation is perhaps more appropriate than the short train or pulse of regular wave approach commonly advocated in the literature for constructing the safe basin [14]. Note that the roll motion model provided by eq. (2) is adequate for modelling ship capsizing, with the stipulation that $a_4<0$, so that the stiffness is of linear-minus-cubic type.

The safe basin is constructed by considering a uniform grid (100×100) of starting values. The grid is defined in the domain $[-1.2(a_3/|a_4|)^{1/2}, 1.2(a_3/|a_4|)^{1/2}] \times [-2(a_3^2/2|a_4|)^{1/2}, 2(a_3^2/|a_4|)^{1/2}]$, which incorporates the area of stable solutions in the undamped free oscillation problem ($a_1=a_2=0$, $M=0$). For each initial condition, eq. (2) is solved by assuming that the largest excitation occurs at $t_0=2T_n$. Numerical investigations have shown that this t_0 is large enough for computing the effect of the whole quasi-impulsive time variation of the excitation. Further, it is small enough for including initial condition influence. Numerical integration is carried out until capsizing occurs or $t=10T_n$. Then, each starting point is marked by a black point, if capsizing has occurred, or by a white point, otherwise.

Fig. 3 shows sequences of safe basins calculated by the increasing amplitude $M(t_0)$ of the excitation. Twelve amplitudes, uniformly distributed over the interval [0,0.45], are assumed. System parameters are $a_1=0.0555$, $a_2=0.01659$, $a_3=0.2227$, $a_4=-0.0694$, as given by Soliman and Thompson [14] for the Gaul ship, while spectral parameters are $k_1=0.97$, $c_1=0.2$, $k_2=3.44$ and $c_2=2$.

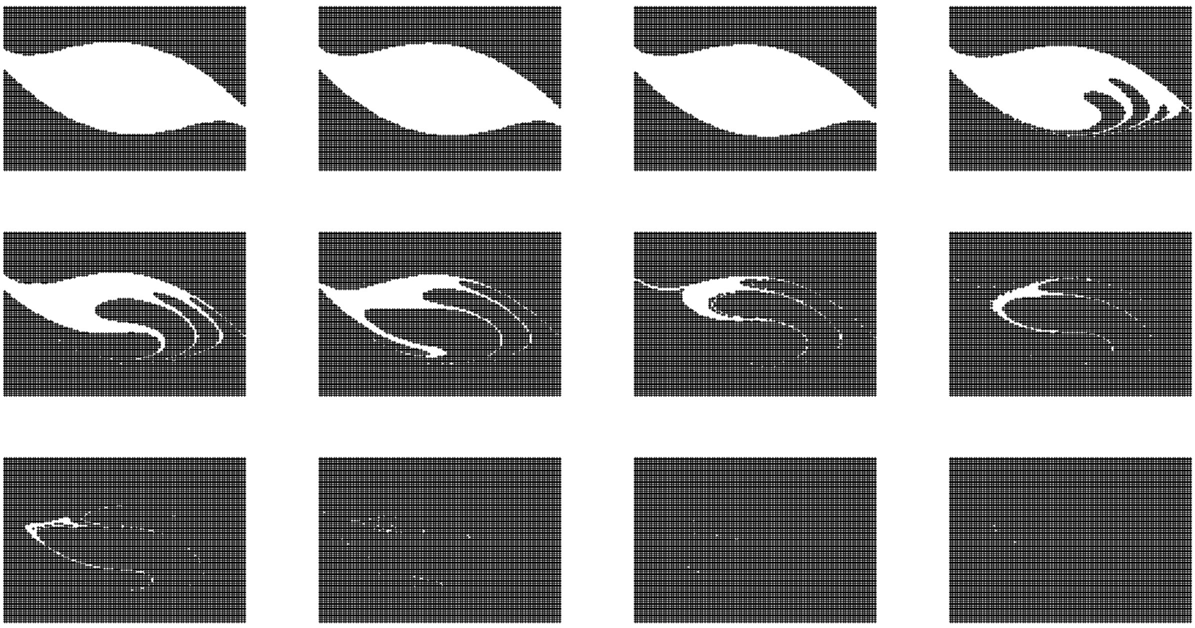

Figure 3: Safe basins obtained by increasing values of $M(t_0)$, uniformly distributed over the interval [0,0.45].

The first basin in fig. 3 (first line, first column) shows the safe points in the free decay case. The successive basins show the progressive erosion of the safe basin area. For smaller amplitudes of the excitation, the erosion results in a striation at the edge of the basin. Then, striations evolve in finger-like zones, that penetrate suddenly in the heart of the safe basin. For quite large amplitudes, the safe basin is totally eroded. The global view produced by the safe basin representation is useful for emphasizing that unsafe operational conditions are not associated only to quite large excitations. Once striations penetrate inside the safe basin, operational conditions are indeed unsafe, despite the limited amplitude of the excitation, because even starting conditions far from the edge of the first basin may relate to capsizing.

4.2 Safety against capsizing measured by integrity curves

A measure of the safety of the system can be derived by integrity curves. The integrity curves are determined by the ratio of the area of the actual safe basin over the area of the safe basin estimated in the free decay case. Obviously, the integrity curves depend both on the system under investigation and on the characteristics of the excitation. In the following, they are represented as a function of the amplitude $M(t_0)$. A key advantage of the proposed approximation is the inclusion of spectral effects on the calculation of integrity curves. This effect is investigated by assuming two spectral shapes of the excitation. Specifically, the power spectral density a), defined by parameters k_1=0.97, c_1=0.2, k_2=3.44, c_2=2.32; the power spectral density b) defined by parameters k_1=0.47, c_1=0.2, k_2=3.44, c_2=2.32. The left panel of Fig. 4 shows both spectra in the frequency domain. They are both single peaked, as is typical for excitations in wind-generated seas, but have different peak frequencies. This fact results in different integrity curves. The right panel of Fig. 4 shows the integrity curves calculated for a system exposed to excitations a) and b). It is seen that they render a synthetic description of the safe basin erosion. Indeed, the cliff-like behaviour of the curves relates to the sudden penetration of unsafe areas into the safe basin. The unsafe operational conditions do not arise gradually. Instead, the ship retains almost all its still water stability up to a certain amplitude of the excitation, and then, it loses dramatically its stability.

The integrity curve utilization in evaluating ship stability relates to the calculation of a threshold P of available safe areas. For instance, if safe operational conditions are related to a threshold P = 10%, it means that the ship is considered safe if "points" in the safe basin are at least 10% of the safe "points" in the still water problem. Thus, for the problem under examination, the amplitude $M(t_0)$ must not exceed 0.32, for the case a), and 0.13, for the case b). This example shows that spectral shapes play indeed a significant role in extreme conditions and even small differences lead to rather significant changes of the critical values of $M(t_0)$.

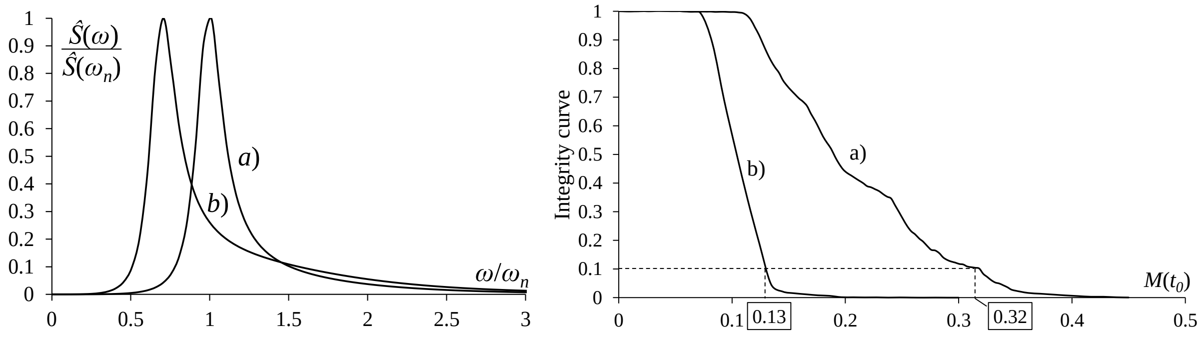

Figure 4: Power spectral density (left panel) and integrity curve (right panel) for a system subject to excitations having spectral parameters: a) k_1=0.97, c_1=0.2, k_2=3.44, c_2=2.32; b) k_1=0.47, c_1=0.2, k_2=3.44, c_2=2.32.

5 CONCLUDING REMARKS

A novel approximate technique for estimating the maximum roll angle of a ship in random waves has been proposed. The technique relies on the quasi-deterministic formulation of the wave exciting moment, which provides a deterministic representation of the realizations of the excitation in the vicinity of a large maximum. The reliability of the proposed formulation has been assessed by pertinent Monte Carlo simulations. It has been shown that the formalism provides a reliable estimate of the maximum roll angle. Further, the proposed technique has the quite desirable feature of been dramatically more efficient vis-à-vis Monte Carlo simulations.

Next, ship stability is investigated by integrity curves measuring safe operational conditions in the space of starting values. In this context, it has been shown that the proposed formalism provides a physically consistent approximation which is preferable to short train or pulse of regular waves commonly utilized as the worst-case scenario. Further, the quasi-determinism based formulation enables a straightforward investigation of the spectral effects on the safe basins and, thus, on the integrity curves.

REFERENCES

[1] R.A. Ibrahim and I.M. Grace, “Modeling of Ship Roll Dynamics and Its Coupling with Heave and Pitch”, *Mathematical Problems in Engineering*, (2010).

[2] G. Bulian, “Nonlinear parametric rolling in regular waves—a general procedure for the analytical approximation of the GZ curve and its use in time domain simulations”. *Ocean Engineering*, **32**(3–4), 309-330 (2005).

[3] V.L. Belenky and N.B. Sevast’ianov, *Stability and safety of ships risk of capsizing*, Jersey City, NJ: Society of Naval Architects and Marine Engineers (2007).

[4] O.M. Phillips, “The theory of wind-generated waves”, *Advances in Hydroscience*, 119-149 (1967).

[5] P.D. Spanos and B.A. Zeldin, “Monte Carlo treatment of random fields: A broad perspective”. *Applied Mechanics Reviews*, **51**(3), 219-237 (1998).

[6] V.L. Belenky et al., “Approaches to rare events in stochastic dynamics of ships”. *Probabilistic Engineering Mechanics*, **28**, 30-38 (2012).

[7] P. Boccotti, “On the highest waves in a stationary Gaussian process”. *Atti Acc. Ligure di Scienze e Lettere*, **38,** 271-302 (1981).

[8] G. Lindgren, “Some Properties of a Normal Process Near a Local Maximum”. *The Annals of Mathematical Statistics*, **41**(6), 1870-1883 (1970).

[9] J.M.T. Thompson and M.S. Soliman, “Fractal Control Boundaries of Driven Oscillators and their Relevance to Safe Engineering Design”. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*, **428**(1874), 1-13 (1990).

[10] J.B. Roberts, J.F. Dunne, and A. Debonos, “Stochastic estimation methods for non-linear ship roll motion”. *Probabilistic Engineering Mechanics*, **9**(1–2), 83-93 (1994).

[11] M. Vasta and J.B. Roberts, “Stochastic parameter estimation of non-linear systems using only higher order spectra of the measured response”. *Journal of Sound and Vibration*, **213**(2), 201-221 (1998).

[12] P.D. Spanos, “Filter approaches to wave kinematics approximation”. *Applied Ocean Research*, **8**(1), 2-7 (1986).

[13] M.B. Allen and E.L. Isaacson, *Numerical Analysis for Applied Science*. Wiley (1998).

[14] M.S. Soliman and J.M.T. Thompson, “Transient and steady state analysis of capsize phenomena”. *Applied Ocean Research*, **13**(2), 82-92 (1991).