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MODELLING FLUID-STRUCTURE INTERACTION IN FLAPPING WINGS THROUGH A COMBINED LATTICE BOLTZMANN-FINITE ELEMENT METHOD

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Parole chiave: Interazione fluido-struttura, Metodo Lattice Boltzmann, Time Discontinuous Galerkin, Decollo di un uccello.

Abstract. *In this paper, the lattice Boltzmann method is used to investigate the behavior of two symmetric flapping wings. First, the theoretical and computational aspects of such method are discussed, as well as its statistical background. Then, findings for rigid and deformable wings are illustrated, showing how this methodology can predict the lift generation due to the motion of the flapping wings.*

Sommario. *In questo documento, il metodo lattice Boltzmann è usato per descrivere il comportamento di due ali simmetriche. Per prima cosa, gli aspetti teorici e computazionali di tale metodo sono discussi insieme al background statistico. Successivamente, i risultati ottenuti per ali rigide e deformabili sono illustrati, mettendo in luce come questa metodologia possa predire la generazione di una forza di lift dovuta al moto delle ali.*

1 INTRODUCTION

A deep understanding of the main mechanical factors regulating the flapping flight of winged insects is extremely important for bio-inspired designing of microscale air vehicles.
Bi-dimensional symmetric flapping flight was recently addressed by Ota et al. in [1]. They assumed the wings as rigid beams and investigated numerically the effect of the Reynolds number in the range 40-200. For high Reynolds numbers (that is greater than 55) they found that asymmetric vortices develop and a lift force is generated, while for low Reynolds numbers (that is lesser than 50) symmetric vortices are stable and no lift force is generated. Moreover, for a certain range of Reynolds numbers (that is between 55 and 110), they observed a bi-stable phenomenon with the wings going upward or downward depending on disturbances.
In this paper, bi-dimensional symmetric flapping flight is investigated by focusing the attention on the effect of wings flexibility. The study is carried out trough a numerical approach recently developed by the authors for fluid-structure interaction problems, that properly couples the Lattice Boltzmann method as fluid solver and the Finite Element methods as structure solver [2,3].
In Section 2, the Lattice Boltzmann method and its statistical background are briefly recalled. In Section 3, the main features of the adopted numerical algorithm are emphasized. Section 4 presents the main results obtained for symmetric flexible flapping wings. Some conclusions are drawn in Section 5.

2 LATTICE BOLTZMANN METHOD

The state of a fluid is quantitatively described through the Navier-Stokes equations, based on the conservation of mass, energy and momentum and on the assumption that the fluid is a continuous material, which is to say that properties are space-filling fields and vary continuously in space and time (i.e. the so-called "continuum hypothesis").
Nevertheless, a fluid is composed of small particles, atoms and molecules, that interact each other, and the fluid motion is ultimately determined by these interactions.
The Lattice Boltzmann Method (LBM) stands in an intermediate (mesoscopic) level of fluids description, in which a statistical molecular-level interpretation of macroscopic fluid dynamics is performed. The fluid composed of a large number of particles is represented in terms of the *probability* (density distribution) $f(\mathbf{r},\mathbf{v},t)$ of finding a given particle at a given position in space, $\mathbf{r}$, and time, *t*, with a given velocity, $\mathbf{v}$. The base equation is the Boltzmann's equation, established already in 1854 for the kinetic theory of ideal gases by Maxwell and Boltzmann, that described how the particle distribution of a diluted fluid changes with time [4,5]:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f = C \,. \tag{1}$$

The above equation states that the free streaming of the particles (left-hand side) equals the changes in the *probability distribution function* $f(\mathbf{r},\mathbf{v},t)$ induced by inter-particle collisions (right-hand side). This statistical approach, although much more economical than the atomistic description, still provides much more information than the continuum description, since the distribution function lives in a six-dimensional space (phase-space) spanned by the particle position $\mathbf{r}$ and velocity $\mathbf{v}$. The LBE is obtained by assuming that at each site the particles can only move along a finite number of directions, thus reducing the velocity space to a handful of discrete values $\mathbf{v} = \boldsymbol{c}_i$, $i=0,\dots n$, and by integrating Equation (1) along these characteristic directions:

$$f_i(\mathbf{r} + \boldsymbol{c}_i \Delta t, t+\Delta t) = f_i(\boldsymbol{c}_i,t)+\Delta t(f_i^{eq} - f_i)/\tau \qquad (2)$$

being Δt the time step. Particle collisions are represented through a relaxation to a local equilibrium, being τ the relaxation time, after the Bathnagar-Gross-Krook (BGK) approximation [6]. The discrete local equilibria are typically given by a second-order expansion in the Mach-number of a local Maxwellian equilibrium, i.e.

$$f_i^{eq}(\mathbf{r},t)=w_i \rho(1+u_a c_{ia}/c_s^2+(u_a u_b Q_{iab})/(2c_s^4)), \qquad (3)$$

where ρ is the density, w_i is a set of weights normalized to unity, $Q_{iab} = c_{ia} c_{ib} - c_s^2 \delta_{ab}$ is the quadrupole projector along the i-th direction of the components c_{ia} and c_{ib} of vectors $\boldsymbol{c}_i$, δ_{ab} is the Kronecker delta, u_a is a component of the fluid velocity $\boldsymbol{u}$, and $c_s^2 = \Sigma_i w_i \boldsymbol{c}_i^2/D$ is the lattice sound speed in D dimensions. Notice that in the above relation Einstein summation convention is adopted. If the set of discrete speeds $\boldsymbol{c}_i$ is chosen with sufficient symmetries to fulfill the basic conservation laws of mass, momentum and energy, the large-scale limit (Chapman-Enskog expansion) of Equation (2) recovers the incompressible, isothermal, Navier-Stokes equations [7], with a kinematic viscosity

$$\nu = c_s^2(\tau - \Delta t/2), \qquad (4)$$

with fluid mass density and velocity determined by the zeroth and first order kinetic moments of the probability distribution functions, as follows:

$$\rho(\mathbf{r},t) = \Sigma_i f_i(\mathbf{r},t), \qquad (5)$$

$$\rho(\mathbf{r},t)\boldsymbol{u}(\mathbf{r},t) = \Sigma_i f_i(\mathbf{r},t)\boldsymbol{c}_i. \qquad (6)$$

In this work we refer to the D2Q9 particles speed model: a standard 9-speed, two-dimensional lattice, with n=8 (including a zero-speed particle), as shown in Figure 1. Apparently, the LBM is only first-order accurate in time. Indeed, the LBM is a first-order scheme for the continuum LBE with a viscosity $\nu = c_s^2 \tau$ but becomes second-order for the same equation, with a shifted viscosity defined in Equation (4). This shift is an issue, since it permits to achieve vanishing low viscosities without taking the relaxation parameter τ and the time step Δt to correspondingly vanishing small values, thereby avoiding the time-step collapse and the ensuing dramatic drop in computational efficiency.

The most attractive features of this probabilistic approach to fluid dynamics are the conceptual and practical simplicity of the computational scheme and the nearly ideal amenability to parallel computing. Moreover, non-locality (streaming) is linear and non-linearity (collision) is local in contrast with the Navier-Stokes equations, in which the transport term is both non-local and non-linear.

For solving fluid-structure interaction problems, the LBM has two further advantages:

1) handling boundary conditions associated with highly irregular geometries by means of elementary mechanical operations on the probability distribution functions (bounce-back, reflections);

2) local availability of fluid pressure p through the ideal equation of state and of stress tensor Σ as linear combinations of the equilibrium and non-equilibrium components of the probability distribution function:

$$p(\mathbf{r},t)=\rho(\mathbf{r},t)\,c_s^2, \tag{7}$$

$$\Sigma(\mathbf{r},t)= -p(\mathbf{r},t)\boldsymbol{I} - (1-\Delta t/(2\tau))\,\Sigma_i(f_i^{eq}- f_i)\boldsymbol{c}_i \otimes \boldsymbol{c}_i, \tag{8}$$

where $\boldsymbol{I}$ is the identity tensor. This means that the force acting on a solid body can be obtained from the flow field configuration with no need of solving a (usually very expensive) Poisson problem and of calculating and interpolating velocity derivatives.

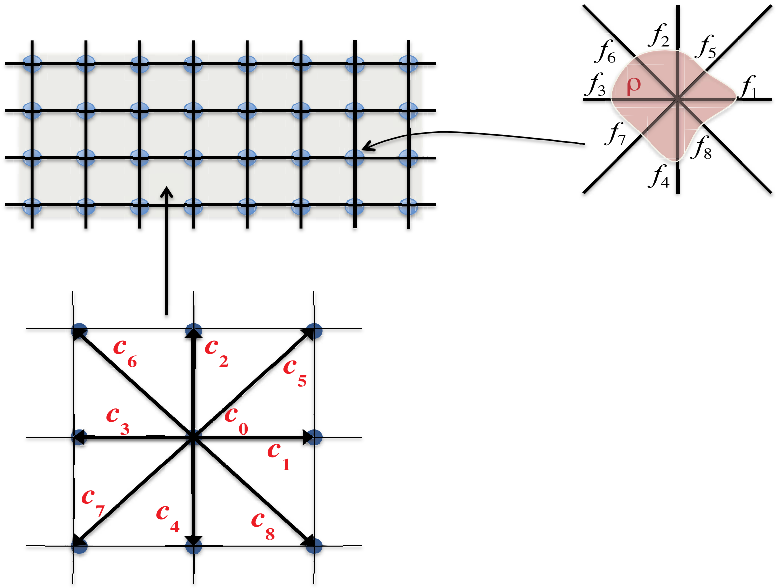

Figure 1: D2Q9 model. Lattice grid, particle distribution function and velocity vectors.

3 FLUID-STRUCTURE INTERACTION

As above assessed, the lattice Boltzmann method is used as the fluid solver. On the other hand, flapping wings are modeled by using the finite element method. In particular, beam elements are employed and large displacements are accounted for by means of the corotational formulation [8,9]. Structural dynamics is integrated by using the Time Discontinuous Galerkin method [10]. If the wings are rigid, the TDG method is used to update the position of the center of mass [11,12], while the boundary condition described in [13,14] is adopted to account for the position of the wings in the fluid domain. The numerical scheme proposed in [13] is used in order to achieve second-order accurate results [14]. An implicit predictor-based coupling algorithm has been implemented [2,3] to transfer information between the fluid and the solid solvers, thus satisfying equilibrium and compatibility conditions at the fluid-structure interface.

4 FLAPPING WINGS

A two-dimensional symmetric flapping wing is immersed in a viscous, incompressible fluid characterized by density ρ and viscosity ν. Wings can travel only in vertical direction. Two different scenarios are investigated: rigid and flexible wings. The wings are represented by two beams with length L connected to a hinge where the mass of the whole set is concentrated, as shown in Figure 2. At the time t, the angular position $\theta(t)$ is given by

$$\theta(t) = \Delta\theta \cos(2\pi t/T), \quad (8)$$

□□□□□ $\Delta\theta$ is the amplitude, T is the period of the harmonic oscillation, and the time-averaged tip velocity is defined as

$$u_{tip}=4L\Delta\theta/T. \quad (9)$$

According to [15], the following parameters corresponding to a butterfly are used: wing mass $3.5\text{x}10^{-6}$ kg, body mass $5.0\text{x}10^{-5}$ kg, hinge-wing distance l=$5.0\text{x}10^{-3}$ m, two-dimensional air density ρ_0=$7.\ 0\text{x}10^{-3}$ kg/m^2. Notice that the total mass is equal to m = $5.7\text{x}10^{-5}$ kg.

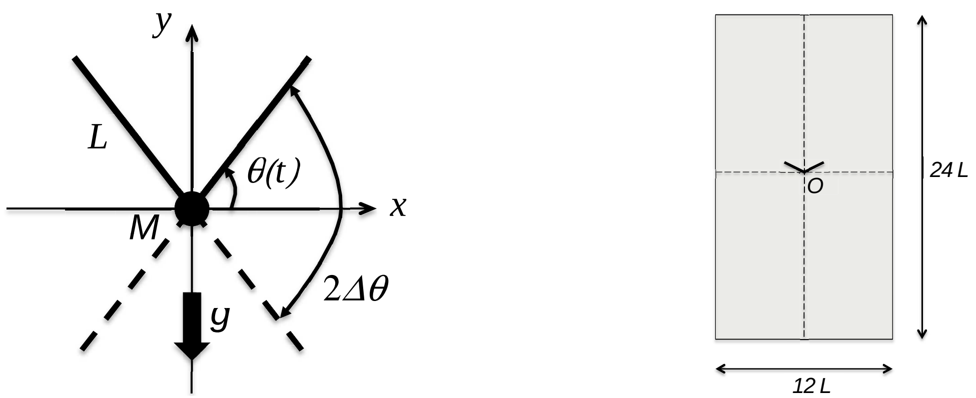

Figure 2: Schematic of the problem definition.

The dimensionless parameters of the problem are: the Reynolds number Re= $u_{tip}\ l/\nu$, the dimensionless mass M, the dimensionless bending stiffness EJ and the Froude number, Fr =$1/g$, where g is the gravity acceleration. The mass is normalized by $\rho_0 l^2$ and the bending stiffness by $\rho_0 l^3 u_{tip}^{\ 2}$.Moreover, in the following the wings position is normalized by L. In order to avoid high velocities in the flow field due to the motion of the wings, the grid dimensions are chosen in such a way that the maximum tangential velocity is less than $c_s/3$. In particular, for Re=40 the grid consists of 1440x720 lattice nodes and the wings are modeled by using 60 elements, while at Re=200 the dimensions are 1920x960 and 80 beam elements are used.

First, the effect of the Reynolds number is discussed. As it is possible to observe in Figure 3, a very close agreement between the present solution and the results obtained in [1] is achieved. Notice that at Re=200 the wings successfully go upward, while at Re=40 they tend to oscillate about a fixed position.

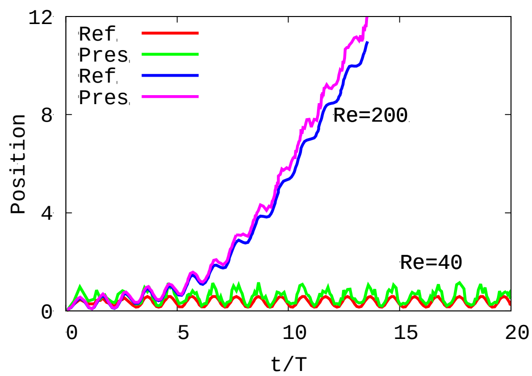

Figure 3: Effect of the Reynolds number.

Then, the effect of wings flexibility is investigated. Figure 4 shows the dimensionless trajectory for different values of the dimensionless bending stiffness *EJ*. Gravity is neglected and the dimensionless mass is $M=9.05$. Various amplitudes are considered: $\Delta\theta=15°$, $\Delta\theta=30°$, $\Delta\theta=46.8°$.

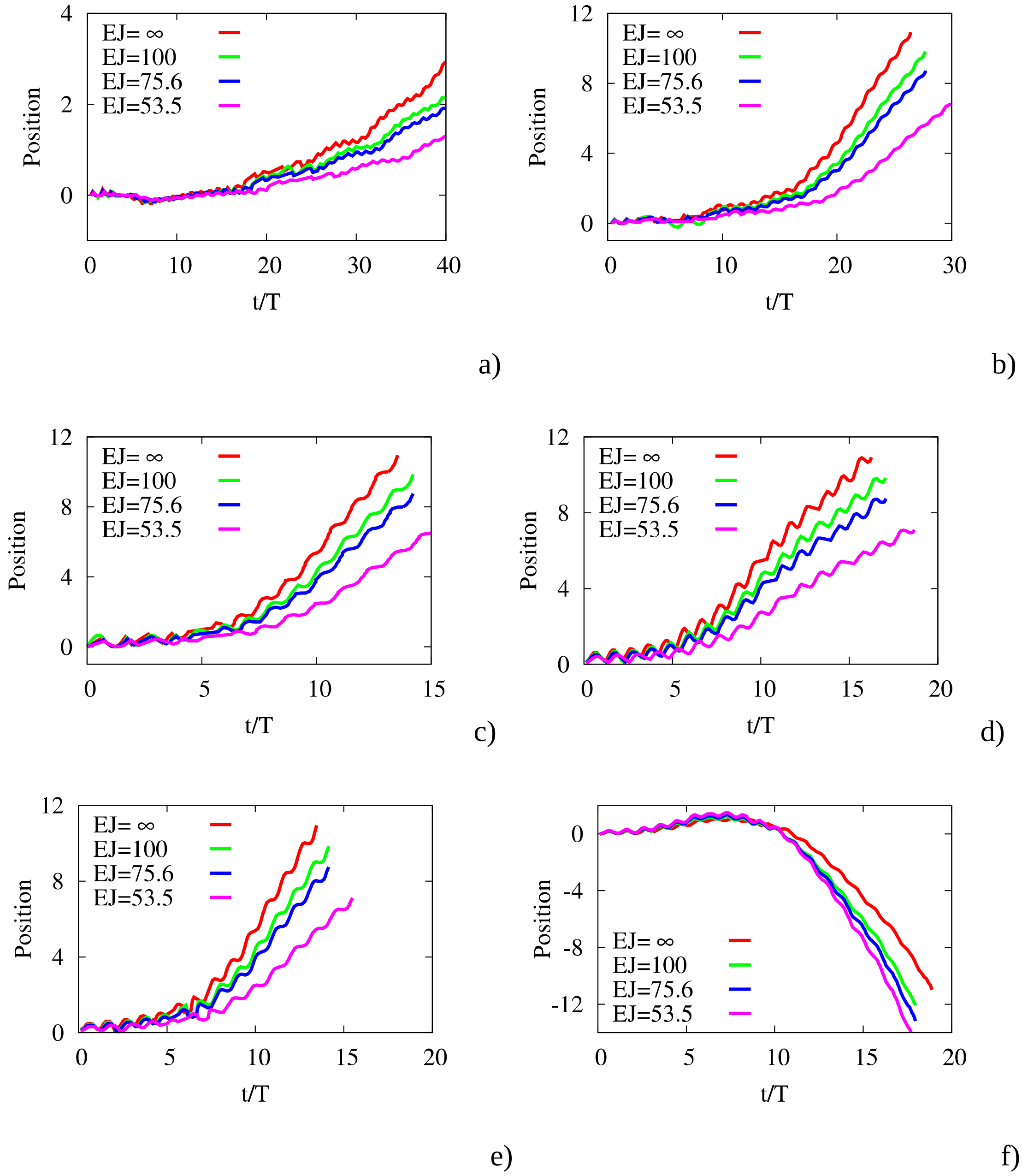

Figure 4: Influence of the dimensionless bending stiffness on the dimensionless position *y/L* of the center of the wings for different amplitudes and masses: a) $\Delta\theta=15°$, b) $\Delta\theta=30°$, c) $\Delta\theta=46.8°$ with $M=9.05$, and d) $M=4.53$, e) $M=9.05$, f) $M=36.2$ with $\Delta\theta=46.8°$.

As we can observe, the larger the amplitude, the faster the take-off is. Moreover, the bending stiffness plays an important role, since the bird goes upward faster for large values of *EJ*. For a certain flapping amplitude, $\Delta\theta=46.8°$, three different values of the dimensionless mass are

used: *M=4.53*, *M=9.05* and *M=36.2*. Also in this case, gravity is neglected and the influence of the mass is shown. In particular, at *M=4.53* and *M=9.05* the wings go successfully upward, whereas at *M=36.2* the trajectory moves in downward direction.
Finally, the effect of the gravity is investigated by varying the Froude number in Figure 6. The dimensionless mass is *M=9.05* and the maximum amplitude is set to $\Delta\theta=45°$. As expected, when the gravitational force increases, the take off becomes arduous.

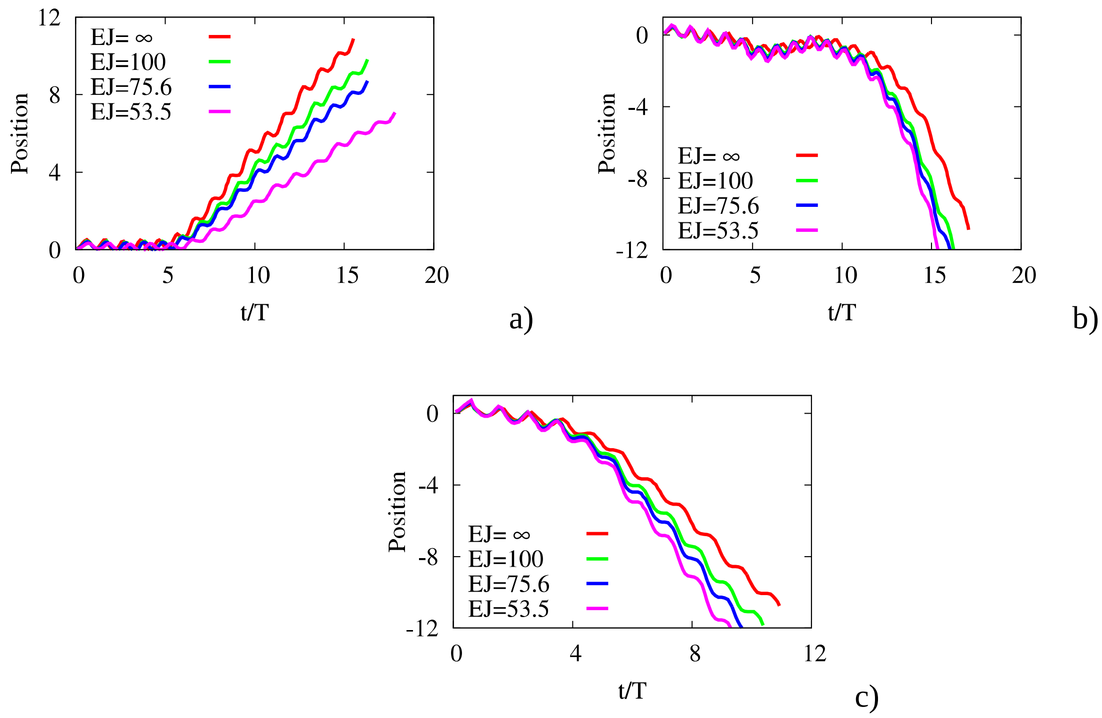

Figure 6: Influence of the dimensionless bending stiffness on the dimensionless position *y/L* of the center of the wings for different Froude numbers: a) *Fr*=8, b) *Fr*=7, c) *Fr*=6.

5 CONCLUSIONS

In this work, the behavior of two symmetric flapping wings immersed in a viscous fluid has been investigated. First, assuming the wings are rigid, the effect of the Reynolds number has been shown. In particular, at *Re*=200 the wings move upward from the original position. Then, the assumption of rigid wings has been removed. The effect of the bending stiffness has been shown for different conditions, i.e. various values of amplitude, mass and Froude numbers. Results show that the lower the bending stiffness, the more difficult the take off is. Such behavior is due to the fact that the energy generated by the motion is partially absorbed by the deformation of the wings, which increases if the bending stiffness decreases.

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