UNIVERSITÀ DEGLI STUDI DI PALERMO

Meccanica dei Materiali e delle Strutture
Vol. 3 (2012), no.3y, pp. 1-8
ISSN: 2035-679X
Dipartimento di Ingegneria Civile, Ambientale, Aerospaziale, Dei Materiali
DICAM

ANALYSIS AND MODELING OF NATURAL FREQUENCY COALESCING IN THE DYNAMIC RESPONSE OF TALL BUILDINGS

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(Ricevuto 10 Giugnpo 2012, Accettato 10 Ottobre 2012)

Key words: Frequency Coalescence, Tall Buildings, Amplitude-dependent Damping.

Parole chiave: Frequenze Coalescenti, Edifici Alti, Smorzamento Dipendente dalla Risposta.

Abstract. *The scope of this paper is to investigate the possibility of defining simplified non-linear dynamic systems that are capable of capturing and reproducing the fundamental natural frequency coalescing phenomenon, which has been observed under certain conditions, in the wind induced response of tall buildings. To this end, a system with two degrees of freedom, representing the two coalescent modes of the building, is considered, while the amplitude-dependent and therefore non-linear damping is investigated as a possible cause of the observed phenomenon. For this system the analytical time-invariant conditions under which the frequency coalescence can occur are firstly identified. Then, stationary exciting functions derived from experimental wind tunnel tests are considered, allowing the direct observation of the phenomenon by means of time-frequency wavelet analysis.*

Sommario. *In questo lavoro viene studiata la possibilità di definire sistemi dinamici semplificati e non lineari capaci di catturare e riprodurre il fenomeno della coalescenza delle frequenze naturali, osservato in particolari condizioni nella risposta eolica di edifici alti. A questo scopo viene introdotto un sistema a due gradi di libertà, rappresentanti i due modi coalescenti dell'edificio, il cui smorzamento non lineare, dipendente dalla risposta, viene studiato quale possibile causa del fenomeno. Per questo sistema vengono innanzitutto identificate le condizioni analitiche tempo-invarianti sotto le quali la coalescenza delle frequenze può manifestarsi. Successivamente, considerando forzanti stazionarie derivate da prove in galleria del vento, il fenomeno è direttamente osservato grazie all'uso di analisi tempo-frequenza mediante trasformate wavelet.*

1 INTRODUCTION

The phenomenon of fundamental natural frequency coalescing has been observed during the analysis of full scale response data obtained as part of a monitoring program on a 246 m tall steel building located in Boston, USA [1]. The response of this building is characterised by coupled fundamental lateral and torsional modes with associated frequencies that have been estimated to be closely spaced. In particular, a time-frequency wavelet analysis of the full scale acceleration response has highlighted a tendency for two of the fundamental vibrational modes to coalesce. This tendency has been observed under certain conditions of wind speed intensity and wind direction [1] but has not yet been fully understood.

This paper investigates the possibility of defining simplified non-linear dynamic systems that are capable of capturing and reproducing the phenomenon of fundamental natural frequency coalescing. In particular, the inherently amplitude dependent, and therefore non-linear, nature of the structural damping is considered as a possible cause. Indeed, this type of behaviour has been observed for the aforementioned building [2], as well as many other tall buildings where full scale response data has been analysed in an attempt to shed light on the characteristics of damping in tall buildings [3]. Numerous investigations have also been carried out in order to understand the underlying mechanisms with the aim of defining appropriate models that are able to reproduce the experimentally observed dependency of the fundamental damping ratios on amplitude [e.g. 5–7].

In order to study the possible influence of amplitude-dependent damping on the structural response, a case study with two degrees of freedom (2-DOF) is here considered, representing two coupled fundamental modes of a building. Firstly, the time-invariant system conditions under which the modal frequencies of the 2-DOF system coalesce are investigated [7–9]. Then, stationary experimentally determined wind forces are assumed as exciting functions of a building, with the fundamental response represented by the 2-DOF system, the nonlinear response of which is investigated for time-variant system conditions conducive to frequency coalescing. Through a time-frequency wavelet-based analysis of the case study response, the frequency coalescing phenomenon is observed for fundamental frequency spacing similar to those reported for buildings where this phenomenon has been observed.

2 THE TWO-DEGREES-OF-FREEDOM SYSTEM AND THE CONDITIONS FOR MODE COALESCENCE

In this section, the behaviour of a simple two degrees of freedom system is investigated with the aim of identifying the time-invariant system conditions under which the observed phenomenon of coalescing frequencies can occur. The 2-DOF system schematizes the first translational and rotational mode of a building.

The two equilibrium equations, written with reference to a system of axes $Oxyz$ with origin located in the centre of stiffness of the system (see figure 1) are:

$$m_x\ddot{x} - a_y m_x \ddot{\theta} + 2m_x\hat{\omega}_x\hat{\xi}_x\dot{x} + k_x x = F_x \tag{1}$$

$$I_\theta\ddot{\theta} - a_y m_x \ddot{x} + 2I_\theta\hat{\omega}_\theta\hat{\xi}_\theta\dot{\theta} + k_\theta\theta = M_z \tag{2}$$

where m_x and I_θ are the mass and moment of inertia, a_y is the eccentricity of the centre of mass with respect to the reference system centre, F_x and M_z are the external force and moment, $\hat{\omega}_x$ and $\hat{\omega}_\theta$ are defined as $\hat{\omega}_x = \sqrt{\frac{k_x}{m_x}}$, $\hat{\omega}_\theta = \sqrt{\frac{k_\theta}{I_\theta}}$, k_x and k_θ define the stiffness of the two degrees of freedom, while the over-dot indicates derivative with respect to time. In a state-space formulation, the previous equilibrium equations can be written as:

$$\dot{\mathbf{y}}(t) = \mathbf{A}\mathbf{y}(t) \tag{3}$$

where $\mathbf{A}$ is defined as:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \tag{4}$$

with

$$\mathbf{M} = \begin{bmatrix} m_x & -a_y m_x \\ -a_y m_x & I_\theta \end{bmatrix},\ \mathbf{C} = \begin{bmatrix} 2m_x\hat{\omega}_x\hat{\xi}_x & \mathbf{0} \\ \mathbf{0} & 2I_\theta\hat{\omega}_\theta\hat{\xi}_\theta \end{bmatrix},\ \mathbf{K} = \begin{bmatrix} k_x & \mathbf{0} \\ \mathbf{0} & k_\theta \end{bmatrix} \tag{5}$$

while $\mathbf{y} = \{x\ \theta\ \dot{x}\ \dot{\theta}\}^T$. As can be seen, the coupling is given by the off-diagonal terms in the mass matrix. The time-invariant eigenvalues and eigenvectors of the 2-DOF system, λ and $\tilde{\mathbf{y}}$, are obtained by solving the complex eigenvalue analysis for the system in equation (3), that is by solving the problem:

$$\mathbf{A}\tilde{\mathbf{y}} = \lambda\tilde{\mathbf{y}} \tag{6}$$

The characteristic equation that allows the calculation of the eigenvalues λ is given by:

$$\begin{aligned} &\lambda^4 - 2B\left(\hat{\xi}_x\hat{\omega}_x + \hat{\xi}_\theta\hat{\omega}_\theta\right)\lambda^3 + B\left(4\hat{\xi}_x\hat{\xi}_\theta\hat{\omega}_x\hat{\omega}_\theta B(1-b) - \hat{\omega}_x^2 - \hat{\omega}_\theta^2\right)\lambda^2 + \\ &+2\hat{\omega}_x\hat{\omega}_\theta B^2(1-b)(\hat{\xi}_x\hat{\omega}_\theta + \hat{\xi}_\theta\hat{\omega}_x)\lambda + B^2\hat{\omega}_x^2\hat{\omega}_\theta^2(1-b) = 0 \end{aligned} \tag{7}$$

where $b = {a_y^2 m_x}/{I_\theta}$ and $B = \dfrac{1}{b-1}$.

It is of interest to the present study to consider the case in which the two conjugate eigenvalue pairs (therefore the two circular frequencies of the coupled and damped system) are identical. In this case, the eigenvalues have the form:

$$\lambda_{1,2} = \lambda_{3,4} = \alpha \pm i\beta \tag{8}$$

where $i = \sqrt{-1}$ and α and β are real numbers. The modal circular frequencies and the damping ratios of this system (neglecting the multiplicity) are:

$$\omega_{1,2} = \sqrt{\alpha^2 + \beta^2} \quad \text{and} \quad \xi_{1,2} = -\frac{\alpha}{\sqrt{\alpha^2 + \beta^2}} \tag{9}$$

The characteristic equation, whose solutions are given by equation (8), is easily demonstrated to be given by:

$$\lambda^4 - 4\alpha\lambda^3 + (6\alpha^2 + 2\beta^2)\lambda^2 - 4\alpha(\alpha^2 + \beta^2)\lambda + (\alpha^2 + \beta^2)^2 = 0 \tag{10}$$

By comparing the coefficients of equations (7) and (10), conditions on time-invariant quantities characterizing the dynamic behaviour of the system can be obtained, that yield coincident eigenvalues:

$$\begin{cases} B\left(\hat{\xi}_x\hat{\omega}_x + \hat{\xi}_\theta\hat{\omega}_\theta\right) = 2\alpha \\ B\left(4\hat{\xi}_x\hat{\xi}_\theta\hat{\omega}_x\hat{\omega}_\theta B(1-b) - \hat{\omega}_x^2 - \hat{\omega}_\theta^2\right) = 6\alpha^2 + 2\beta^2 \\ \hat{\omega}_x\hat{\omega}_\theta B^2(1-b)\left(\hat{\xi}_x\hat{\omega}_\theta + \hat{\xi}_\theta\hat{\omega}_x\right) = -2\alpha(\alpha^2 + \beta^2) \\ B^2\hat{\omega}_x^2\hat{\omega}_\theta^2(1-b) = (\alpha^2 + \beta^2)^2 \end{cases} \tag{11}$$

In particular, having chosen the quantities m_x, I_θ, k_x (or $\hat{\omega}_x$), for any couple of values $\Delta\hat{\xi} = \hat{\xi}_x - \hat{\xi}_\theta = \Delta\xi^*$ and $\hat{\xi}_\theta = \xi^*$ it is possible to find $\hat{\omega}_\theta$ (therefore k_θ), b (therefore a_y), α and β (therefore $\omega_{1,2}$ and $\xi_{1,2}$) satisfying the conditions of equation (11). Figure 2 shows the values obtained through the aforementioned procedure (with $e_y = \frac{a_y}{D}$), for m_x = 3.50e4 ton, I_θ = 1.40e7 ton·m^2, $\hat{\omega}_x = 0.9425$ and $\xi^* =$ 0.2% while $\Delta\xi^*$ varies between 0 and 8%.

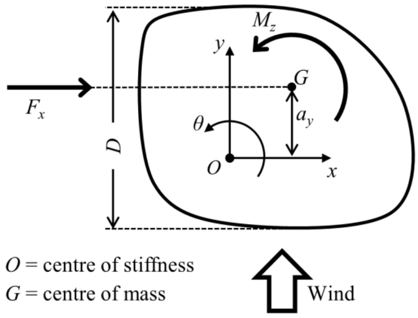

Figure 1: The 2-DOF system.

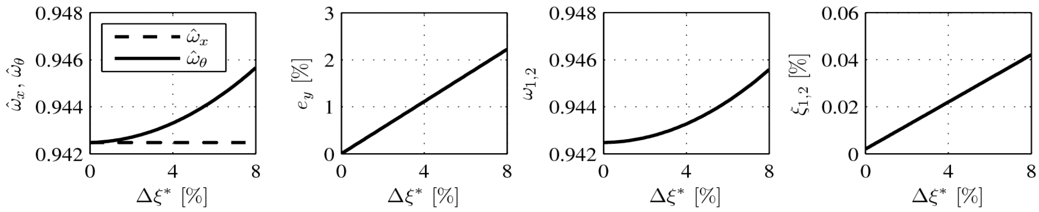

Figure 2: Variations of the quantities $\hat{\omega}_\theta$, e_x, $\omega_{1,2}$, $\xi_{1,2}$, satisfying the conditions of equation (11) for $\xi^* = 0.2\%$ and $\Delta\xi^*$ varying from 0 to 8%.

It is also interesting to observe how the damping and modal properties of the system, $\xi_{1,2}$ and $\omega_{1,2}$, vary with $\Delta\hat{\xi}$ when all other quantities are fixed and calibrated in order to generate mode coalescence for $\Delta\hat{\xi} = \Delta\xi^*$ and $\hat{\xi}_\theta = \xi^*$. This case is of particular interest to the problem under investigation, where the role of amplitude dependent damping on the possible coalescence of the modes is to be studied. Indeed, when such a system is subject to external excitation, it is possible that the natural frequencies will vary and possibly coalesce due to a variation in the instantaneous damping levels (time-variant system characteristics) and therefore their difference. This will occur, however, only if the two conditions $\Delta\hat{\xi} = \Delta\xi^*$ and $\hat{\xi}_\theta = \xi^*$ are contemporarily satisfied. Figure 3, for example, illustrates how the modal damping ratios and circular frequencies vary with $\Delta\hat{\xi}$ in the case that the characteristics of the system are calibrated to respect the conditions (11) for $\Delta\xi^* =$ 4.4% and $\xi^* =$ 0.2%.

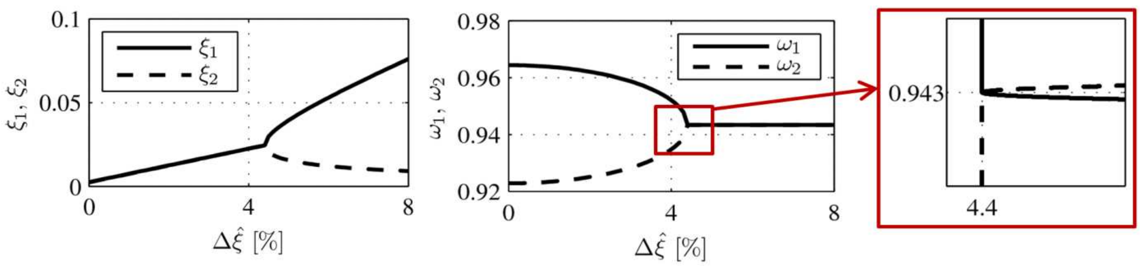

Figure 3: Variations of $\xi_{1,2}$ and $\omega_{1,2}$ with $\Delta\hat{\xi}$ for a system calibrated with $\Delta\xi^* = 4.4\%$ and $\xi^* = 0.2\%$.

By observing figure 3 it should be noticed that for any value of $\Delta\hat{\xi}$ greater than $\Delta\xi^*$, even if the eigenvalues do not perfectly coincide, they still remain extremely close. Also, if the mechanical characteristics of the building do not perfectly match the characteristics that would ensure the satisfaction of the conditions of equation (11), but are close, a sudden convergence of the eigenvalues can still be observed for $\Delta\hat{\xi} \cong \Delta\xi^*$. This is an important observation in light of the practical application of this study, as it illustrates how frequency coalescence could occur for conditions proximate to the ideal case.

3 DAMPING MODEL

The dependency of the damping ratio on the amplitude of the building response has been reported and modelled by numerous authors [4–6]. In particular in [4] it was suggested that the amplitude dependent expected damping ratio for the s th mode in tall buildings may be estimated through a power law as:

$$\hat{\xi}_s = B\left(\bar{x}_s/_H\right)^{\gamma} \quad (1)$$

where B and γ are dimensionless parameters, H is the height of the building and $\bar{x}_s$ is the response level. This law was defined based on the assumption that the bulk of damping in structures is generated by friction between structural and/or non-structural components and that the number of dissipative friction sources indefinitely increases with amplitude. This model was chosen in this study for describing the amplitude dependent nature of damping in tall buildings due to its simplicity and the large quantity of experimental data suggesting its general validity for regions of moderate amplitude response [3]. The extension of the considerations made in this paper to more sophisticated damping models would be immediate.

4 CASE STUDY AND RESULTS

4.1 Description of the case study

The case study here considered for investigating the influence of the amplitude dependent damping on natural frequency coalescence is a 2-DOF system schematically representing a 180 m tall building, as illustrated in figure 4. The building is a rectangular prism with cross-section of 36×72 m. Wind tunnel tests with measurements of pressures have been carried out on models of the aforementioned building by the Tokyo Polytechnic University (TPU) and the data has been published as part of the *TPU Wind Pressure Database* [10]. The total length of the signals is 32 s, with a sampling frequency of 1000 Hz; the mean wind speed during the tests at the building model top was 11 m/s while the model scale was 1:360. By integration of the experimental data, two time histories of the forcing functions representing the translational (across-wind) and torsional wind excitation have been obtained. In particular, the contribution of the pressures acting over the top half of the building surface (see figure 4) was considered.

Two different sets of mechanical characteristics of the system, summarized in table 1, are considered, calibrated by means of the conditions of equation (11), in order to have for Case 1 frequency coalescence for $\xi^* =$ 0.25% and $\Delta\xi^* =$ 4.25% while for Case 2 $\xi^* =$ 0.25% and $\Delta\xi^* =$ 3.4%. The mass and inertial terms have been estimated considering a mass per unit volume of the building of 150 kg/m^3. For the sake of completeness, the two modal frequencies obtained through classical modal analysis carried out for the two systems are also indicated in table 1. The damping laws describing the amplitude-dependency, and therefore the time-variant system characteristics, are characterized by the parameters $B = 0.042$, $\gamma = 0.25$ for the translational degree of freedom and $B = 0.003$, $\gamma = 0.05$ for the rotational degree of freedom and are represented in figure 4.

When the damping satisfies both the conditions $\hat{\xi}_\theta = \xi^*$ and $\Delta\hat{\xi} = \Delta\xi^*$, the complex eigenvalues of the system will coincide as will do the modal frequencies and associated damping ratios. As previously mentioned, however, the frequency coalescence can also be observed for conditions within the proximity of the case which gives perfect matching of the two eigenvalues.

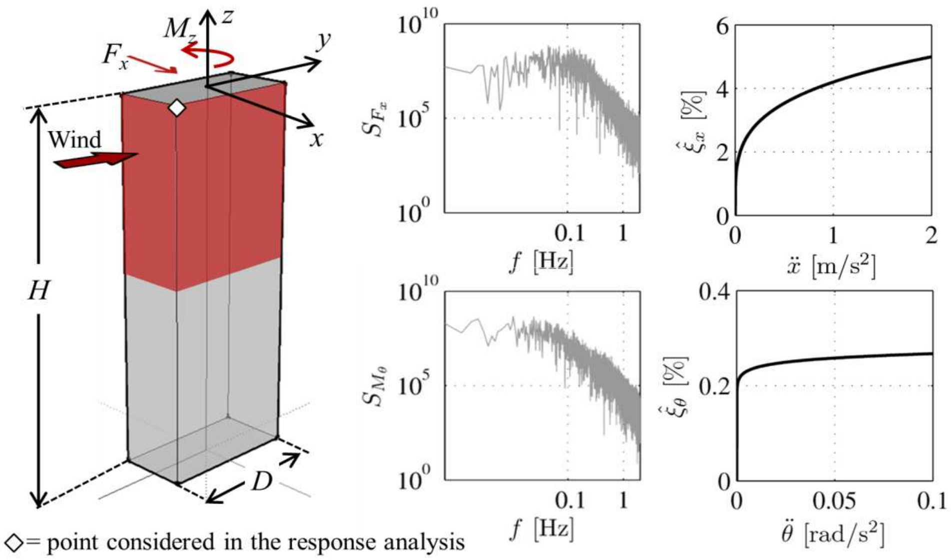

Figure 4: Case study: building schematized with a 2-DOF system (left), experimentally determined forcing function spectra (centre) and assumed damping laws (right).

	$\overline{m}_x$ [ton]	$\overline{I}_\theta$ [ton·m^2]	$\hat{\overline{\omega}}_x$ [1/rad]	$\hat{\overline{\omega}}_\theta$ [1/rad]	$\overline{e}_y$ [%]	$\overline{f}_1$ [Hz]	$\overline{f}_2$ [Hz]
Case 1	34992	1.40e7	0.9425	0.9434	1.18	0.1357	0.1478
Case 2	34992	1.40e7	0.9425	0.9431	0.94	0.1368	0.1464

Table 1: Mechanical characteristics of the 2-DOF case study, for Case 1 and Case 2

4.2 Results

In order to investigate the possibility that the instantaneous variation in the fundamental damping ratios could cause the frequencies at which the non-linear system vibrates to shift and in particular coalesce, a time-frequency analysis using the continuous Morlet wavelet transform was carried out on the x -direction corner acceleration. Through the analysis of the ridge of the wavelet scalogram and the instantaneous wavelet spectra, the evolution of any time-varying frequency features that may be in the response signal can be identified. In particular, for the case in question, a central frequency of 8 Hz was chosen, yielding a frequency resolution of 0.0084 Hz (4 standard deviations of the wavelet frequency window).

Figures 5 and 6 present the results of the aforementioned analysis for Cases 1 and 2 in terms of the ridges of the wavelet scalograms as well as the instantaneous wavelet spectra corresponding to the numbered time instants (indicated by the dashed lines in figures 5 and 6). It is immediately obvious that, under appropriate conditions, the non-linear 2-DOF oscillator will experience vibrational frequency coalescing. Both figure 5 and 6 clearly illustrate the role of response amplitude in determining the frequency response of the system. In particular,

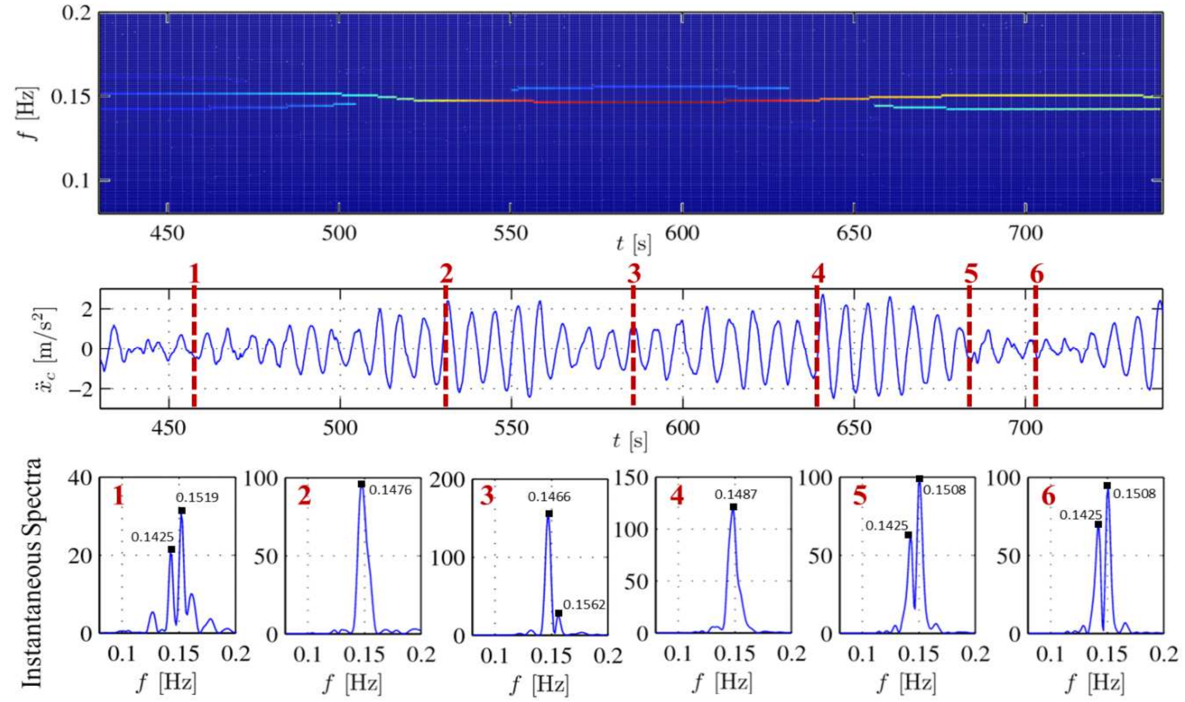

Figure 5: Frequency coalescence observed for Case 1 calibrated with $\Delta\xi^* =$ 4.25%, $\xi^* =$ 0.25% and wind speed of 72.9 m/s; from the top: ridge of the wavelet scalogram, time history of the response and instantaneous spectra.

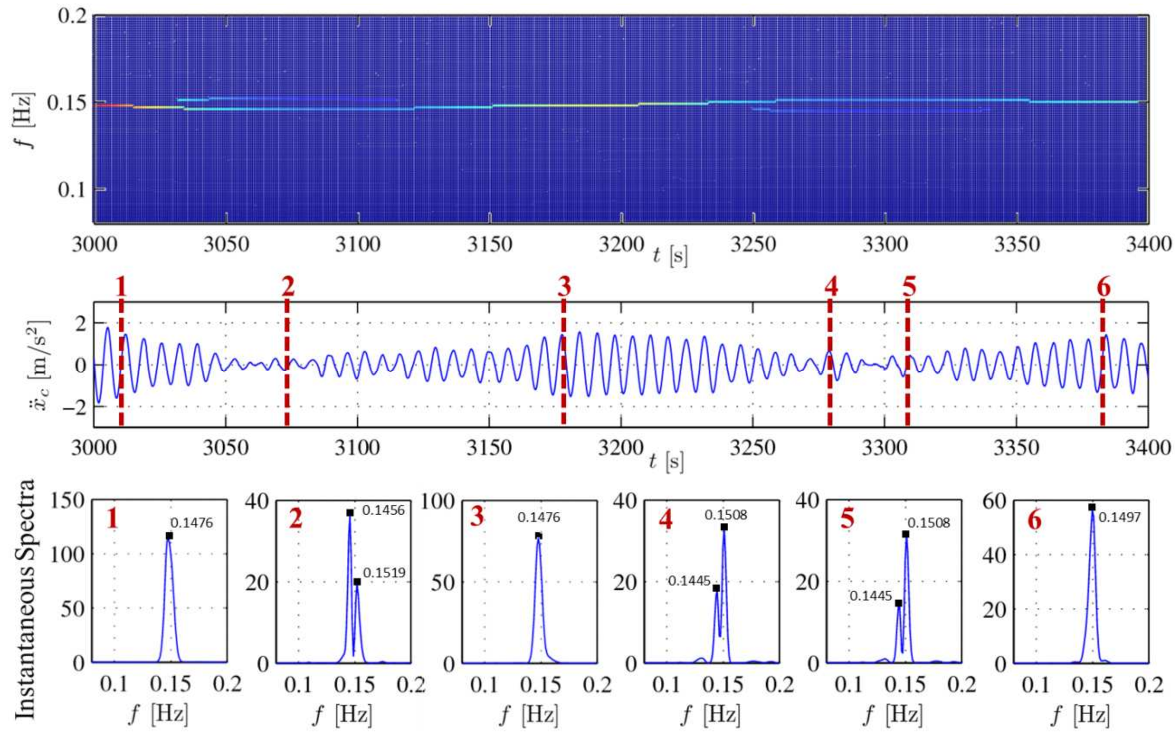

Figure 6: Frequency coalescence observed for Case 2 calibrated with $\Delta\xi^* = 3.4\%$, $\xi^* =$ 0.25% and wind speed of 49.5 m/s; from the top: ridge of the wavelet scalogram, time history of the response and instantaneous spectra.

for the low amplitude region (dashed lines 1,5,6 for figure 5 and 2,4,5 for figure 6) the instantaneous wavelet spectra illustrates how the system is responding in two distinct frequencies, while for high amplitude response (lines 2,4 for figure 5 and 1,3,6 for figure 6) the instantaneous spectra show the presence of a single dominate mode with frequency between the two low amplitude frequencies. It is interesting to observe that the non-linear response features presented in figures 5 and 6 show extremely similar characteristics to those reported in [1] concerning the full scale response of a 246 m steel tall building.

5 CONCLUSIONS

In this paper the possibility that the inherently amplitude dependent nature of damping in tall buildings is investigated as a possible cause of a peculiar phenomenon that has been observed in the full scale response of such structures where the fundamental natural frequencies have been seen to coalesce when the response amplitude increases. In order to investigate this possibility, the analytical time-invariant conditions under which such a phenomenon may occur are first identified for a 2-DOF system schematically representing the coupled fundamental building response. Through a time-frequency wavelet analysis carried out on the calibrated response dependent (time-variant and non-linear) 2-DOF system excited by stationary experimentally determined forcing functions, the phenomenon of frequency coalescing is clearly reproduced indicating the influence of amplitude dependent damping in the frequency response of tall buildings. The extension of this work to more complex damping models is natural.

Acknowledgments: Support was in part provided by the Global Center of Excellence at Tokyo Polytechnic University, funded by MEXT.

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