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DISCRETE VARIABLE STRUCTURAL DESIGN OF FRAMES SAFE AGAINST BUCKLING

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Parole chiave: Variabili discrete, Progetto ottimale, Carichi dinamici, Stabilità.

Abstract. *The present paper is devoted to the formulation of a special minimum volume design problem of elastic perfectly plastic frames subjected to different combinations of fixed and seismic loads. The optimal structure must behave elastically for fixed loads, shakedown for serviceability conditions and prevent the instantaneous collapse for suitable combinations of fixed and seismic loadings. The design variables are represented by defined discrete sets. Moreover, element buckling and P-Delta effects are considered. Different numerical applications conclude the paper.*

Sommario. *Il lavoro che si presenta è rivolto ad una particolare formulazione del problema di progetto di minimo volume di telai costituiti da materiale a comportamento costitutivo elastico perfettamente plastico e soggetti a diverse combinazioni di carichi fissi ed azioni sismiche. La struttura è progettata in modo da esibire un comportamento elastico sotto gli assegnati carichi fissi, da adattarsi elasticamente ai carichi di esercizio e da scongiurare il collasso istantaneo per la combinazione di carichi fissi ed elevati carichi sismici. Si assume l'ipotesi che le variabili di progetto appartengano ad opportuni insiemi discreti e si tiene conto della snellezza degli elementi strutturali, così come degli effetti P-Delta. Si effettuano diverse applicazioni numeriche.*

1 INTRODUCTION

The structural optimization problems are very often formulated as search for the minimum structural weight which substantially provides the minimum cost to be suffered for the structure construction.
On the other side, the choice of the design admissibility conditions is very complex and specific of the particular problem which must be formulated. These conditions are substantially represented by inequalities identifying one or more limit behaviour related to the material and/or to the structure. If reference is made to elastic plastic structures, the limit

conditions can characterize: the purely elastic limit behaviour; the shakedown limit behaviour; the plastic shakedown and/or of the incremental collapse limit behaviours beyond which the structure suffers an instantaneous collapse.
In addition, further admissibility conditions can be requested. These conditions depending on the load condition and on the displacement structural response are related with possible P-Delta effects and/or with possible buckling of some structural element.
In the last decades several efforts have been devoted to the study of the optimal design of structures under quasi-static loads as well as dynamic actions and the fundamental results are reported in several books and papers (see, e.g., [1-15]).
Aim of the present paper is to propose the formulation of a discrete variables minimum volume design problem for elastic perfectly plastic frame structures subjected to a combination of fixed and seismic actions is proposed. The optimal structure is constrained to behave in a purely elastic manner for the assigned fixed loads, to respect the elastic shakedown limit in serviceability conditions and to prevent the instantaneous collapse for ultimate seismic load conditions; the P-Delta effects are considered. Furthermore it is required to prevent the risk of element buckling for all the above described load combinations. The dynamic response is obtained by utilizing an appropriate modal technique referring to the response spectrum defined by the Italian code. Several numerical applications are effected by utilizing an harmony search algorithm (see, e.g., [16-17]).

2 THE FRAME STRUCTURE: SCHEME AND ELASTIC RESPONSE

In order to appropriately describe the proposed optimal design formulation, some fundamentals mainly regarding the definition of some appropriate model both for the frame structure and for the acting loads must be introduced.

Let us consider a classical plane frame with n_b Navier type beam elements and n_N structure nodes, each characterized by three degrees of freedom. The following quantities are defined: $\boldsymbol{u}$, frame nodal displacement vector of order $3 \cdot n_N$; $\boldsymbol{F}$, frame nodal force vector with the same order of $\boldsymbol{u}$; $\boldsymbol{d}$, element nodal displacement vector of order $6 \cdot n_b$; $\boldsymbol{Q}$, generalized stress vector of order $6 \cdot n_b$ evaluated at the extremes of the elements; $\boldsymbol{Q}^*$, perfectly clamped element generalized stress vector analogous to $\boldsymbol{Q}$.

The static linear elastic analysis problem for the plane frame is given as follows:

$$\boldsymbol{d} = \boldsymbol{C}\boldsymbol{u} \tag{1a}$$

$$\boldsymbol{Q} = \boldsymbol{D}\boldsymbol{d} + \boldsymbol{Q}^* \tag{1b}$$

$$\tilde{\boldsymbol{C}}\boldsymbol{Q} = \boldsymbol{F} \tag{1c}$$

where $\boldsymbol{C}$ is the compatibility matrix with order $6 \cdot n_b \times 3 \cdot n_N$, its transpose $\tilde{\boldsymbol{C}}$ the equilibrium matrix, and $\boldsymbol{D}$ the frame internal stiffness matrix with order $6 \cdot n_b \times 6 \cdot n_b$. The solution to problem (1) is given by:

$$\boldsymbol{u} = \boldsymbol{K}^{-1}\boldsymbol{F}^* \tag{2a}$$

$$\boldsymbol{Q} = \boldsymbol{D}\boldsymbol{C}\boldsymbol{u} + \boldsymbol{Q}^* = \boldsymbol{D}\boldsymbol{C}\boldsymbol{K}^{-1}\boldsymbol{F}^* + \boldsymbol{Q}^* \tag{2b}$$

In equations (2) $\boldsymbol{K} = \tilde{\boldsymbol{C}}\boldsymbol{D}\boldsymbol{C}$ is the frame external square stiffness matrix of order

$3 \cdot n_N \times 3 \cdot n_N$ and $\boldsymbol{F}^* = \boldsymbol{F} - \tilde{\boldsymbol{C}}\boldsymbol{Q}^*$ is the equivalent nodal force vector. Making reference to the seismic actions, let us consider the relevant frame as a flexural plane frame just subjected to an horizontal ground acceleration $a_g(t)$, where the masses are concentrated at each node. The structure is modelled as a Multi-Degree-Of-Freedom (MDOF) one and the dynamic equilibrium equations can be written in the following form:

$$\boldsymbol{M}\,\ddot{\boldsymbol{u}}(t) + \boldsymbol{B}\,\dot{\boldsymbol{u}}(t) + \boldsymbol{K}\,\boldsymbol{u}(t) = \boldsymbol{f}(t), \tag{3}$$

being $\boldsymbol{f}(t) = -\boldsymbol{M}\boldsymbol{\tau}\, a_g(t)$, $\boldsymbol{M}$ and $\boldsymbol{B}$ mass and damping matrices and $\boldsymbol{\tau}$ the influence vector. If the interest is focused only to the case of undulatory dynamic effect, it is possible to operate a partition and reordering of the above described matrices and vectors and to write the free vibration equations as:

$$\begin{pmatrix} \boldsymbol{M}_{tt} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{pmatrix} \begin{pmatrix} \ddot{\boldsymbol{u}}_t(t) \\ \ddot{\boldsymbol{u}}_r(t) \end{pmatrix} + \begin{pmatrix} \boldsymbol{K}_{tt} & \boldsymbol{K}_{tr} \\ \boldsymbol{K}_{rt} & \boldsymbol{K}_{rr} \end{pmatrix} \begin{pmatrix} \boldsymbol{u}_t(t) \\ \boldsymbol{u}_r(t) \end{pmatrix} = \begin{pmatrix} \boldsymbol{f}_t(t) \\ \boldsymbol{0} \end{pmatrix}, \tag{4}$$

where $\boldsymbol{u}_t$ is the vector collecting the structure node horizontal displacements and $\boldsymbol{u}_r$ the vertical displacements and the rotations of the nodes. Equation (4) can be usefully rewritten in the following form:

$$\boldsymbol{M}_{tt}\,\ddot{\boldsymbol{u}}_t(t) + \boldsymbol{K}_{tt}\,\boldsymbol{u}_t(t) + \boldsymbol{K}_{tr}\,\boldsymbol{u}_r(t) = \boldsymbol{f}_t(t), \tag{5a}$$

$$\boldsymbol{K}_{rt}\,\boldsymbol{u}_t(t) + \boldsymbol{K}_{rr}\,\boldsymbol{u}_r(t) = \boldsymbol{0}. \tag{5b}$$

The solution of the equation (5b) is given by:

$$\boldsymbol{u}_r(t) = -\boldsymbol{K}_{rr}^{-1}\boldsymbol{K}_{rt}\,\boldsymbol{u}_t(t). \tag{6}$$

Substituting equation (6) into equation (5a) it is obtained:

$$\boldsymbol{M}_{tt}\,\ddot{\boldsymbol{u}}_t(t) + \boldsymbol{K}_c\,\boldsymbol{u}_t(t) = \boldsymbol{f}_t(t), \tag{7}$$

which represents the condensed equation of motion for free vibration and where $\boldsymbol{K}_c = \boldsymbol{K}_{tt} - \boldsymbol{K}_{tr}\boldsymbol{K}_{rr}^{-1}\boldsymbol{K}_{rt}$ is the condensed stiffness matrix.

Equation (7) can be solved with a classical modal technique according to the initial conditions $\boldsymbol{u}_t(0) = \boldsymbol{0}$, $\dot{\boldsymbol{u}}_t(0) = \boldsymbol{0}$. The dynamic characteristics of the structural behaviour are identified in terms of natural frequencies as well as damping coefficients. The following coordinate transformation is adopted:

$$\boldsymbol{u}_t(t) = \boldsymbol{\Phi}\boldsymbol{z}(t) \tag{8}$$

being $\boldsymbol{z}(t)$ the modal displacement vector and $\boldsymbol{\Phi}$ the so-called modal matrix, normalized with respect to the mass matrix and whose columns are the eigenvectors of the undamped structure, given by the solution to the following eigenproblem:

$$\boldsymbol{K}_c^{-1}\boldsymbol{M}_{tt}\,\boldsymbol{\Phi} = \boldsymbol{\Phi}\,\boldsymbol{\Omega}^{-2} \tag{9a}$$

$$\tilde{\boldsymbol{\Phi}}\,\boldsymbol{M}_{tt}\,\boldsymbol{\Phi} = \boldsymbol{I} \tag{9b}$$

$$\tilde{\boldsymbol{\Phi}}\,\boldsymbol{K}_c\,\boldsymbol{\Phi} = \boldsymbol{\Omega}^2 \tag{9c}$$

In equations (9a,c), besides the already known symbols, $\boldsymbol{I}$ represents the identity matrix while $\boldsymbol{\Omega}^2$ is a diagonal matrix listing the square of the natural frequencies of the structure.

Once the modal matrix $\boldsymbol{\Phi}$ has been determined, the structure can be defined as a classically-damped one if $\tilde{\boldsymbol{\Phi}} \boldsymbol{C}_d \boldsymbol{\Phi} = \boldsymbol{\Xi}$ is a diagonal matrix whose typical non zero element Ξ_{jj} is equal to $2\zeta_j \omega_j$, being ω_j and ζ_j the j^{th} natural frequency and the j^{th} damping coefficient, respectively. The equation of motion in the modal space can be written as:

$$\ddot{\boldsymbol{z}}(t) + \boldsymbol{\Xi}\, \dot{\boldsymbol{z}}(t) + \boldsymbol{\Omega}^2 \boldsymbol{z}(t) = \boldsymbol{g}(t) \tag{10}$$

where $\boldsymbol{g}(t) = \tilde{\boldsymbol{\Phi}} \boldsymbol{f}_t(t)$.

The solution of the decoupled system of equations (10) together with the corresponding initial conditions provides the complete structural response in terms of horizontal displacements in the modal space. Once these displacements are known the corresponding ones in the nodal space are obtained by means of equation (8), the remaining ones being determined by means of equation (6).

Now, according with the guidelines of the most international codes, and in particular with the Italian one, seismic loadings have to be evaluated for two different main conditions: the serviceability conditions and the exceptional (high intensity) one. Therefore, we now assume that the actions are represented by three different appropriate combinations of the above referred loads each of which related to different limit conditions. The first combination is characterized by the presence of the fixed loads $\boldsymbol{F}_0^*$; the second combination is defined as the superimposition of appropriate reduced fixed loads $\boldsymbol{F}_{0e}^*$ and low seismic actions related to the response spectrum S_d^S (serviceability conditions); the third combination is characterized by the superimposition of $\boldsymbol{F}_{0e}^*$ and high intensity seismic actions related to the response spectrum S_d^I (ultimate conditions).

In the above defined combinations, $\boldsymbol{F}_{0e}^*$ is a special combination of gravitational loads as prescribed by the referenced code, S_d^S and S_d^I are the response spectra related to serviceability and instantaneous collapse conditions, respectively.

Clearly, since the design problem under investigation is a minimum volume search one, the structural geometry is not definitely known a priori. Therefore, let the typical ν^{th} element geometry be fully described by the m components of the vector $\boldsymbol{t}_\nu$ $(\nu = 1,2,...,n_b)$ so that $\tilde{\boldsymbol{t}} = \left[\tilde{\boldsymbol{t}}_1, \tilde{\boldsymbol{t}}_2, ..., \tilde{\boldsymbol{t}}_\nu, ..., \tilde{\boldsymbol{t}}_{n_b}\right]$ represents the $n_b \times m$ supervector collecting all the design variables..

3 THE OPTIMAL DESIGN PROBLEM

Let us consider the plane frame structure as above described constituted by elastic perfectly plastic material. Let it be subjected to fixed loads and perfect cyclic dynamic (seismic) loads identifying three different load combinations as previously described. The structure is required to remain elastic when no seismic actions occur, to exhibit a shakedown behaviour in serviceability conditions and to prevent the instantaneous collapse for high level seismic loadings.

Furthermore, dealing with structures constituted by slender elements, the risk of buckling as well as the P-Delta effects are considered.

In order to take into account the so-called P-Delta effects the bending moments acting on the structural nodes must be increased by the bending effect produced by the axial forces acting on the pillars times the drifts at each storey. Furthermore, an approximate approach is utilized in order to account the buckling effect (see, [18]).

Therefore, following all the previous statements and remarks, the minimum volume design problem formulation can be written as follows:

$$\min_{\left(t_k, \boldsymbol{u}_0, \boldsymbol{u}_{tj}^S, \boldsymbol{u}_{tj}^I, \boldsymbol{u}_j^S, \boldsymbol{u}_j^I, \boldsymbol{Y}_0^S, \boldsymbol{Y}_{0i}^I\right)} V \tag{17a}$$

$$t_k \in T_k\,, \quad \left(k = 1,2,......,n_b \times m\right) \tag{17b}$$

$$\boldsymbol{Q}_0 = \boldsymbol{DCu}_0 + \boldsymbol{Q}_0^*\,, \quad \boldsymbol{Ku}_0 - \boldsymbol{F}_0^* = \boldsymbol{0}\,, \tag{17c}$$

$$\boldsymbol{u}_{tj}^S = \boldsymbol{\Phi}_j \frac{\tilde{\boldsymbol{\Phi}}_j \boldsymbol{M}_{tt} \boldsymbol{\tau}\, S_d^S\left(T_j\right)}{\omega_j^2}\,, \quad \boldsymbol{Q}_j^S = \boldsymbol{DCu}_j^S\,, \quad Q_\ell^S = \sqrt{\sum_j \sum_k \rho_{kj}\, Q_{k\ell}^S Q_{j\ell}^S}\,, \tag{17d}$$

$$\boldsymbol{u}_{tj}^I = \boldsymbol{\Phi}_j \frac{\tilde{\boldsymbol{\Phi}}_j \boldsymbol{M}_{tt} \boldsymbol{\tau}\, S_d^I\left(T_j\right)}{\omega_j^2}\,, \quad \boldsymbol{Q}_j^I = \boldsymbol{DCu}_j^I\,, \quad Q_\ell^I = \sqrt{\sum_j \sum_k \rho_{kj}\, Q_{k\ell}^I Q_{j\ell}^I}\,, \tag{17e}$$

$$\boldsymbol{\varphi}^E \equiv \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p\left(\boldsymbol{Q}_0 + \boldsymbol{Q}_0^{(P\Delta)}\right) - \boldsymbol{R} \le \boldsymbol{0}\,, \tag{17f}$$

$$\boldsymbol{\varphi}_i^S \equiv \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p\left(\boldsymbol{Q}_{0e} + \boldsymbol{Q}_{0e}^{(P\Delta)}\right) + \left(-1\right)^i \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p\left(\boldsymbol{Q}^S + \boldsymbol{Q}^{S(P\Delta)}\right) - \boldsymbol{SY}_0^S - \boldsymbol{R} \le \boldsymbol{0}\,, \quad \boldsymbol{Y}_0^S \ge \boldsymbol{0}\,, \tag{17g}$$

$$\boldsymbol{\varphi}_i^I \equiv \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p\left(\boldsymbol{Q}_{0e} + \boldsymbol{Q}_{0e}^{(P\Delta)}\right) + \left(-1\right)^i \tilde{\boldsymbol{N}}\tilde{\boldsymbol{G}}_p\left(\boldsymbol{Q}^I + \boldsymbol{Q}^{I(P\Delta)}\right) - \boldsymbol{SY}_{0i}^I - \boldsymbol{R} \le \boldsymbol{0}\,, \quad \boldsymbol{Y}_{0i}^I \ge \boldsymbol{0}\,, \tag{17h}$$

$$\boldsymbol{\varphi}_{cr}^E \equiv \boldsymbol{G}_{cr}\left(\boldsymbol{Q}_0 + \boldsymbol{Q}_0^{(P\Delta)}\right) - \frac{\boldsymbol{P}_{cr}}{\eta} \le \boldsymbol{0}\,, \tag{17i}$$

$$\boldsymbol{\varphi}_{icr}^S \equiv \boldsymbol{G}_{cr}\left(\boldsymbol{Q}_{0e} + \boldsymbol{Q}_{0e}^{(P\Delta)}\right) + \left(-1\right)^i \boldsymbol{G}_{cr}\left(\boldsymbol{Q}^S + \boldsymbol{Q}^{S(P\Delta)}\right) - \frac{\boldsymbol{P}_{cr}}{\eta} \le \boldsymbol{0}\,, \tag{17j}$$

$$\boldsymbol{\varphi}_{icr}^I \equiv \boldsymbol{G}_{cr}\left(\boldsymbol{Q}_{0e} + \boldsymbol{Q}_{0e}^{(P\Delta)}\right) + \left(-1\right)^i \boldsymbol{G}_{cr}\left(\boldsymbol{Q}^I + \boldsymbol{Q}^{I(P\Delta)}\right) - \frac{\boldsymbol{P}_{cr}}{\eta} \le \boldsymbol{0}\,, \tag{17k}$$

where equations (17g,h,j,k) hold for $i = 1,2$ while $j = 1,2,.....,n_{sm}$, being n_{sm} the number of structural modes and $\ell = 1,2,.....,6 \cdot n_b$.

In equations (17b) t_k $\left(k = 1,2,......,n_b \times m\right)$ are all the design variables defined in the related discrete domains T_k. In equations (17c,d,e) $\boldsymbol{u}_0$ and $\boldsymbol{Q}_0$, $\boldsymbol{u}_j^S = \boldsymbol{E}\left|\boldsymbol{u}_{tj}^S \quad -\boldsymbol{K}_{rr}^{-1}\boldsymbol{K}_{rt}\boldsymbol{u}_{tj}^S\right|^T$ and $\boldsymbol{Q}_j^S$, $\boldsymbol{u}_j^I = \boldsymbol{E}\left|\boldsymbol{u}_{tj}^I \quad -\boldsymbol{K}_{rr}^{-1}\boldsymbol{K}_{rt}\boldsymbol{u}_{tj}^I\right|^T$ and $\boldsymbol{Q}_j^I$ are the purely elastic response to the assigned full fixed loads, to the serviceability seismic loads related to the j^{th} structural mode, to the high intensity seismic loads related to the j^{th} structural mode, respectively, in terms of structure node displacements and element node generalized stresses, being E an appropriate matrix correctly reordering the node displacements.

In equations (17f,g,h) $\boldsymbol{\varphi}^E$, $\boldsymbol{\varphi}_i^S$ and $\boldsymbol{\varphi}_i^I$ are the plastic potential vectors related to the purely elastic limit (apex *E*), to the elastic shakedown limit (apex *S*) and to the instantaneous collapse limit (apex *I*), respectively. $\boldsymbol{Y}_0^S$ and $\boldsymbol{Y}_{0i}^I$ are the fictitious plastic activation intensity vectors related to the elastic shakedown limit and to the impending instantaneous collapse. In addition: $\tilde{\boldsymbol{N}}$ is the matrix of the external normals to the elastic domain; $\tilde{\boldsymbol{G}}_p$ is an appropriate equilibrium matrix which applied to element node generalized stresses provides the same stresses acting upon the element plastic nodes; $\boldsymbol{Q}_0^{(P\Delta)}$ is the vector collecting the element node generalized stresses response related to the P-Delta effect considering acting just the full fixed loads; $\boldsymbol{Q}_{0e} = 0.8\boldsymbol{Q}_0$ is the elastic stress response vector to reduced fixed loads and $\boldsymbol{Q}_{0e}^{(P\Delta)}$ is the analogous of $\boldsymbol{Q}_0^{(P\Delta)}$ but related to the reduced fixed loads; $\boldsymbol{Q}^{S(P\Delta)}$ and $\boldsymbol{Q}^{I(P\Delta)}$ are the analogous of $\boldsymbol{Q}_0^{(P\Delta)}$ but related to seismic serviceability conditions and high level seismic loads, respectively. $-\boldsymbol{S}$ is a time independent symmetric matrix which transforms plastic activation intensities into plastic potentials and $\boldsymbol{R}$ is the relevant plastic resistance vector. Furthermore, equations (17i,j,k) represent the admissibility conditions with respect to the buckling of the pillars related to the three described load combinations; $\boldsymbol{G}_{cr}$ is a condensation matrix able to extract just the axial stress of the pillars; $P_{cr,i} = 1.265\, EI_{min,i} \big/ H_i^2$ is the critical load related to the ith pillar; η is an appropriate safety factor (see [18]). Problem (17) is a quite wide one and it can be utilized for different special applications. Actually, disregarding constraints (17i,j,k) and neglecting the terms affected by the P-Delta effect a standard optimal design is searched, namely a minimum volume one without any constraint accounting for the slenderness of the elements.

4 NUMERICAL APPLICATIONS

The optimal design of the plane steel frames plotted in Fig. 1a has been obtained referring to the formulation previously proposed. At first, the optimal design problem (17) has been solved searching for the standard optimal structure. The frame is constituted by rectangular box cross section elements (Fig. 1b) with $b = 200\,\text{mm}$ and $h = 400\,\text{mm}$, and constant thickness t variable in $T\{4,5,...,40\,\text{mm}\}$. Furthermore, $L_1 = 600\,\text{cm}$, $L_2 = 400\,\text{cm}$ and $H = 600\,\text{cm}$, Young modulus $E = 21\,\text{MN}\big/\text{cm}^2$, yield stress $\sigma_y = 23.5\,\text{kN}\big/\text{cm}^2$.

Two rigid perfectly plastic hinges are located at the extremes of the elements, considered to be elastic, and an additional hinge is located in the middle point of the longer beams (Fig. 1c). The interaction between bending moment M and axial force N has been taken into account. In Fig. 1d the dimensionless rigid plastic domain of the typical plastic hinge is plotted in the plane ($N\big/N_y, M\big/M_y$), being N_y and M_y the yield generalized stress corresponding to N and M, respectively. The structure is subjected to a fixed uniformly distributed vertical load, $q_0 = 50\,\text{kN/m}$ and to seismic actions. We assume that the seismic masses located in the relevant structure nodes are equal for each floor, $m_1 = 12.23\,\text{kN}\cdot\text{s}^2/\text{m}$, $m_2 = 20.39\,\text{kN}\cdot\text{s}^2/\text{m}$, $m_3 = 8.15\,\text{kN}\cdot\text{s}^2/\text{m}$ (Fig. 1a). The response spectra for serviceability conditions (up-crossing probability in the lifetime 81%) and instantaneous collapse (up-crossing probability in the lifetime 5%) are those corresponding to Palermo, with a soil type B, life time 100 years and class IV. An harmony search algorithm has been utilized [16-17]. The obtained results are reported in Table 1.

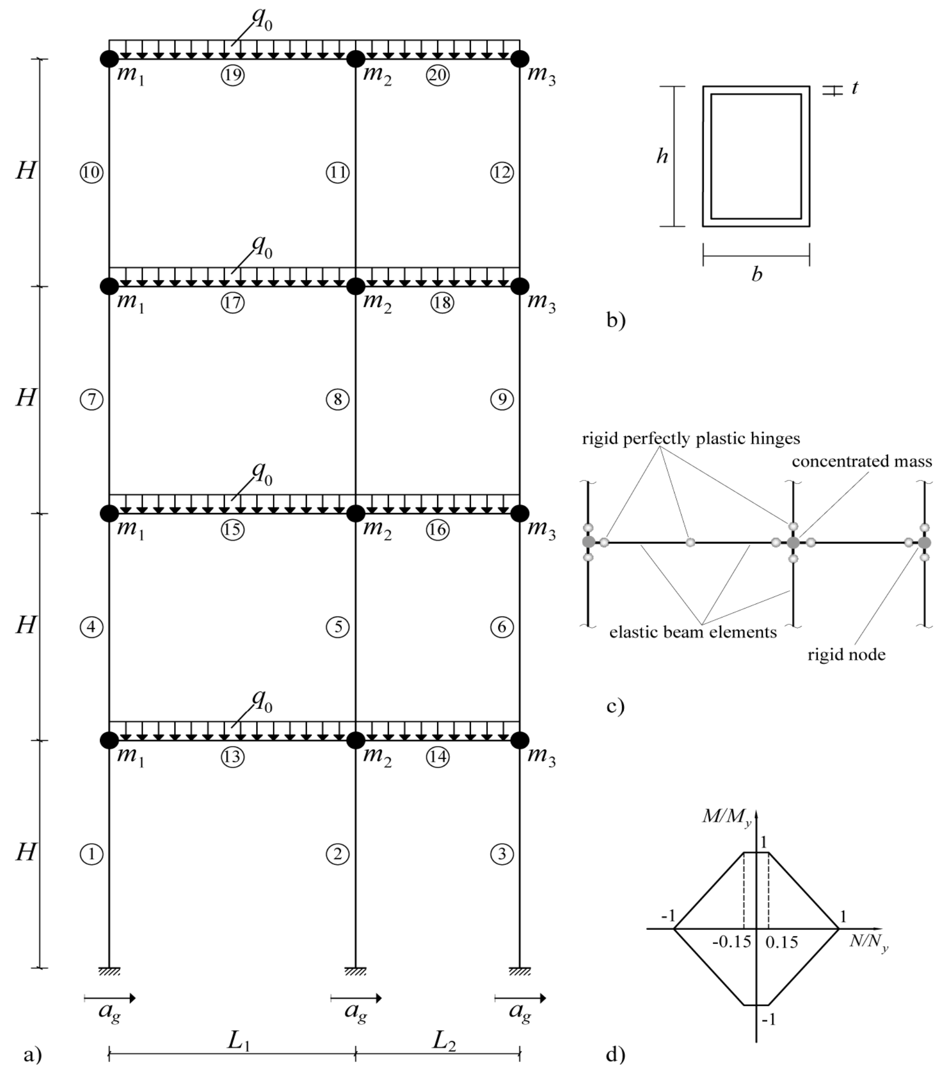

Figure 1: Four floor flexural steel frame: a) geometry and load conditions;
b) typical rectangular box cross section of the elements;
c) structural scheme of the relevant beams;
d) rigid plastic domain of the typical plastic hinge.

Volume: 1.778										
El.	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
t	11	30	20	9	21	14	8	13	9	5
El.	***11***	***12***	***13***	***14***	***15***	***16***	***17***	***18***	***19***	***20***
t	10	4	18	39	21	20	15	10	5	7

Table 1: Standard design volume (m^3) and thicknesses (mm).

The same frame plotted in Fig. 1 has been studied but taking appropriately into account the element slenderness. In particular, the minimum volume optimal design has been searched by considering alternatively element buckling ($\eta = 1.5$), P-Delta effects and both. The results are reported in Tables 2, 3 and 4.

Volume: 1.933										
El.	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
t	40	23	11	10	23	16	11	13	6	9
El.	***11***	***12***	***13***	***14***	***15***	***16***	***17***	***18***	***19***	***20***
t	9	4	36	28	13	24	19	7	5	4

Table 2: Safe buckling design volume (m^3) and thicknesses (mm).

Volume: 1.833										
El.	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
t	9	33	25	14	23	10	6	15	11	9
El.	***11***	***12***	***13***	***14***	***15***	***16***	***17***	***18***	***19***	***20***
t	5	4	20	39	22	21	7	10	7	4

Table 3: Safe P-Delta design volume (m^3) and thicknesses (mm).

Volume: 1.968										
El.	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***
t	18	24	29	16	31	8	9	19	9	11
El.	***11***	***12***	***13***	***14***	***15***	***16***	***17***	***18***	***19***	***20***
t	9	4	23	29	17	21	12	15	7	5

Table 4: Safe buckling/P-Delta design volume (m^3) and thicknesses (mm).

The features of the obtained designs can be interpreted by the relevant Bree diagrams plotted in Fig. 2a-f.

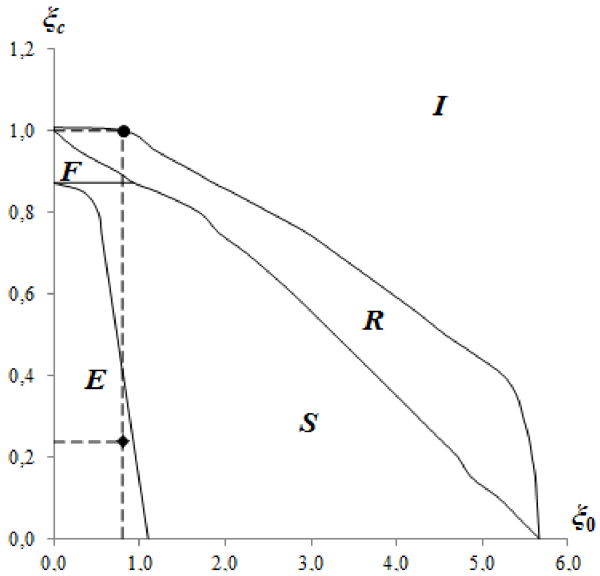

Figure 2a: Bree diagram of the standard optimal design.

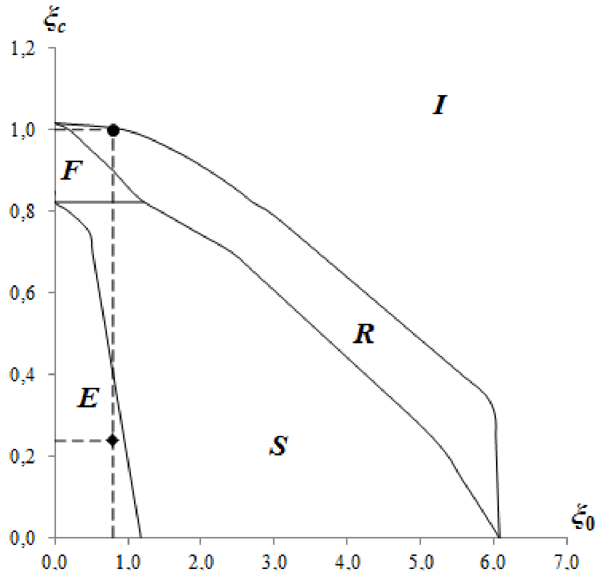

Figure 2b: Bree diagram of the safe buckling optimal design.

As it is easy to observe, as expected, the optimal structures does not violate in any case the imposed safety limit behaviours for the prescribed load combinations, even with same margins due to the chosen discrete range of the design variables. However, they exhibit a dangerous condition of ratchetting even for cyclic load multipliers lower than the prescribed one. By analysing the referenced Bree diagrams it suffices to impose $\xi_c \geq 0.88$ for determining a very dangerous incremental collapse condition. In other papers (see, e.g., [19]) the same authors faced the cited problem proposing different approaches in order to improve the safety structural behaviour; in the present study such a problem is disregarded focusing the attention just to the problem of the element buckling.

On the other side, it is worth noticing the importance of considering the P-Delta effects, especially for the standard design; actually, (Fig. 2e) disregarding such effect the design is unable to satisfy the prescribed safety condition on the instantaneous collapse for high intensity seismic loads.

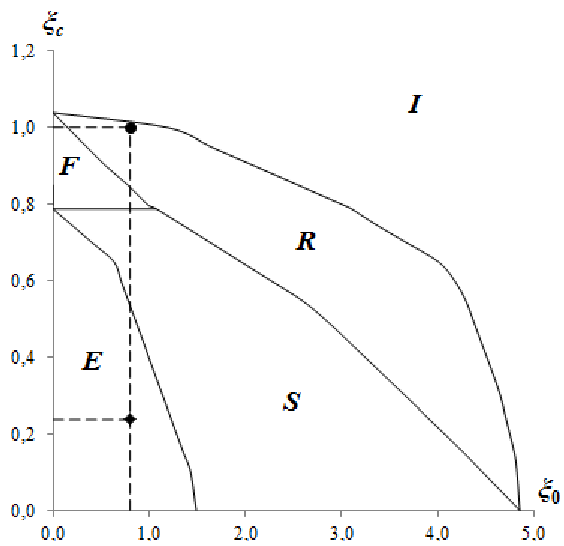

Figure 2c: Bree diagram of the safe P-Delta optimal design.

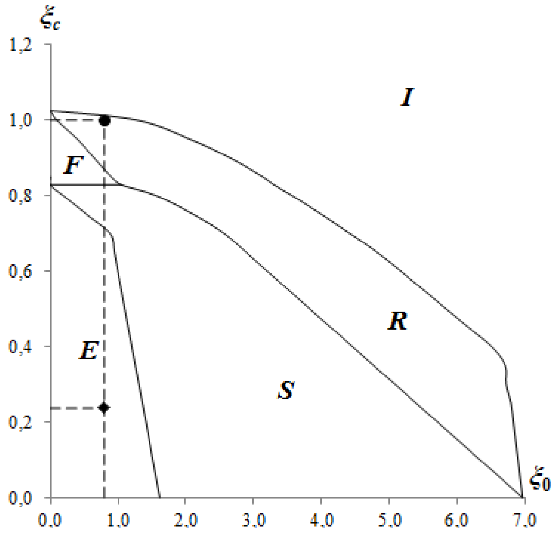

Figure 2d: Bree diagram of the safe buckling/P-Delta optimal design.

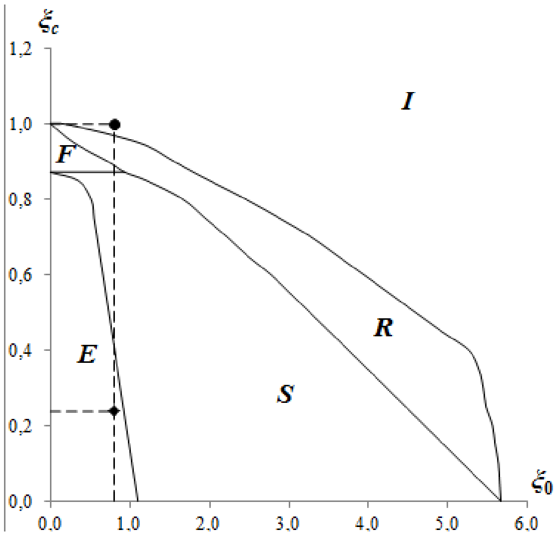

Figure 2e: Bree diagram of the standard design considering P-Delta effect.

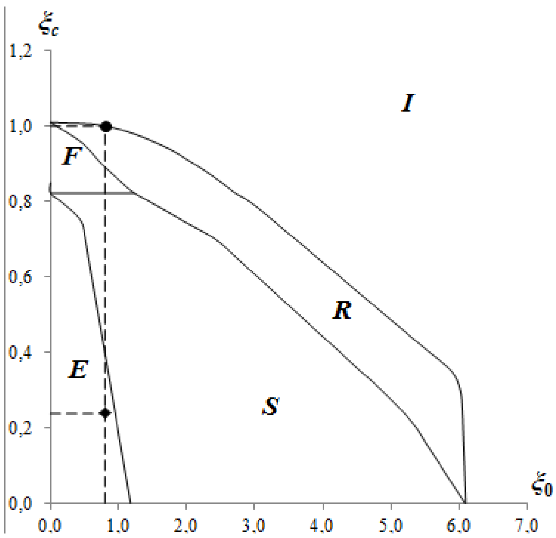

Figure 2f: Bree diagram of the safe buckling design considering P-Delta effect.

5 CONCLUSIONS

The present paper has been devoted to the minimum volume design of plane frames constituted by elastic perfectly plastic material and subjected to suitably defined load combinations characterized by the simultaneous presence of fixed loads and seismic actions. The element thicknesses have been defined as discrete design variables. Three different load combinations have been considered: the basic load combination, constituted by the solely assigned fixed loads; the serviceability load combination, defined as the combination of reduced fixed loads and low seismic actions; the ultimate limit load combination, defined as the combination of suitably fixed loads and high intensity seismic actions. Correspondingly, three resistance limits have been considered: the purely elastic limit, the elastic shakedown limit and the instantaneous collapse limit, besides further special limits on the element buckling.

A four floor plane steel frame has been investigated. At first a standard optimal design problem has been solved, namely disregarding the buckling constraints and the P-Delta effects. Subsequently, the same problem has been solved but introducing the P-Delta effects and/or the constraints on buckling. The features of the obtained structures have been interpreted by the related Bree diagrams.

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