$\mathbf{FA}\&\mathbf{QP}~\mathbf{2023}$

Program and Book of Abstracts

| | Monday 5 | Tuesday 6 | Wednesday 7 | Thursday 8 | Friday 9 |
|-------------|--------------|-----------------|--------------|---------------|---------------|
| 9.00-9.30 | Registration | | | | |
| 9.30-10.15 | Opening | A. Fring | J.P. Gazeau | D. Krejcirik | Y. Joglekar |
| 10.15-11.00 | K. Schmudgen | A. Mostafazadeh | M. Gadella | R. Carlone | N. Hatano |
| 11.00-11.30 | Coffee break | Coffee break | Coffee break | Coffee break | Coffee break |
| 11.30-12.15 | M. Frank | F. Ciccarello | A. Oliaro | A. Teta | Aiena/Triolo |
| 12.15-12.45 | S. Ivkovic | G. Garello | L.Goube | G. Farmakis | G. Bellomonte |
| 13.00-14.30 | Lunch | Lunch | Lunch | Lunch | Lunch |
| 14.30-15.15 | P. Balazs | M. Reboiro | | S. Kuzel | |
| 15.15-16.00 | Z. Mouayn | G. Vinti | GUIDED TOUR | J. Barnett | |
| 16.00-16.30 | Coffee break | Coffee break | | Coffee break | |
| 16.30-17.00 | A. Othman | A. Travaglini | | R. Ukena | |
| 17.00-17.30 | O. Atlasiuk | H. Inoue | | M. Dalla Riva | |
| 17.30-18.00 | | M. E. Griseta | | A. Nastasi | |

19.30-24.00

Social Dinner

List of abstracts

| 1. OthmanAbad | 17. Naomichi Hatano |
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| 2. Pietro Aiena/Salvatore Triolo | 18. Hiroshi Inoue |
| 3. Olena Atlasiuk | 20. Stefan Ivkovic |
| 4. Peter Balazs | 21. Yogesh Joglekar |
| 5. Giorgia Bellomonte | 22. David Krejcirik |
| 6. Jacob Barnett | 23. Sergiusz Kużel |
| 7. Raffaele Carlone | 24. Ali Mostafazadeh |
| 8. Francesco Ciccarello | 25. Zouhair Mouayn |
| 9. Matteo Dalla Riva | 26. Antonella Nastasi |
| 10. George Farmakis | 27. Alessandro Oliaro |
| 11. Michael Frank | 28. Marta Reboiro |
| 12. Andreas Fring | 29. Konrad Schmüdgen |
| 13. Manuel Gadella | 30. Alessandro Teta |
| 14. Gianluca Garello | 31. Arianna Travaglini |
| 15. Jean Pierre Gazeau | 32. Riko Ukena |
| 16. Maria Elena Griseta | 33. Gianluca Vinti |

Application of generalized Drazin-Riesz invertible operators to abstract singular differential equations

Othman Abad¹, Joint work with: **Hassane Zguitti**¹ ¹Department of Mathematics, Sidi Mohamed Ben Abdellah University, Fez, Morocco

The notion of generalized Drazin-Riesz invertible operators was pioneered by S. C. Zivković-Zlatanović and M. D. Cvetković [3]. In this talk, we give a formulation of generalized Drazin-Riesz inverses found in [1], which is related to spectral sets developed by T.D. Tran [2], also, we present the characterization of bounded generalized Drazin-Riesz invertible operators [1], and we apply the notion of generalized Drazin-Riesz invertibility to solve abstract singular differential equations [1].

- O. Abad and H. Zguitti : A note on the generalized Drazin-Riesz invertible operators. Ann. Funct. Anal. 12, Article number: 55 (2021)
- T. D. Tran : Spectral sets and the Drazin inverse with applications to second order differential equations. Applications of Mathematics, 47 (1) 1-8 (2002).
- [3] S. C. Zivković-Zlatanović and M. D. Cvetković : Generalized Kato-Riesz decomposition and generalized Drazin-Riesz invertible operators. Linear and Multilinear. Algebra, 65 (6), 1171-1193, (2017)

Local spectral theory and Weyl's theorem

P. Aiena and S. Triolo

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H. Weyl in 1908 proved that a self-adjoint operator on a Hilbert space has some relevant spectral properties. Later these properties have been observed for Toeplitz operators. More recently it has been proved that there are several important classes of operators that satisfy these properties. An operator which has this spectral picture is said to satisfy Weyl's theorem. In this talk we give a survey of this theory, emphasizing the role that have the local spectral theory.

Solvability and continuity in a parameter of solutions of differential systems with generic boundary conditions

Olena Atlasiuk

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We study linear systems of ordinary differential equations on a finite interval with the most general (generic) inhomogeneous boundary conditions in Sobolev spaces. These boundary problems include all known types of classical and numerous nonclassical conditions. The latter may contain derivatives of integer and fractional order, which may exceed the order of the differential equation.

We investigate the characteristic of solvability of inhomogeneous boundary-value problems, prove their Fredholm properties, and find the indices, the dimensions of the kernel, and the cokernel of these problems. These results are illustrated by examples. Moreover, we obtained the necessary and sufficient conditions for continuity in the parameter of solutions to the introduced boundary-value problems in the Sobolev spaces. We supplemented our result with a two-sided estimate of the error of the solution via its discrepancy. Some applications of these results to the solutions of multipoint boundary-value problems are also presented.

- Atlasiuk O. M.: Limit theorems for solutions of multipoint boundary-value problems with a parameter in Sobolev spaces. Ukrain. Math. J. 72 n.8, 1175–1184 (2021)
- [2] Atlasiuk O. M., Mikhailets V. A.: Fredholm one-dimensional boundary-value problems with parameter in Sobolev spaces. Ukrain. Math. J. 70 n.11, 1677–1687 (2019)

Frame Theory: the functional analytical foundation for acoustics, quantum physics and machine learning

Peter Balazs¹

¹Acoustics Research Institute, Austrian Academy of Sciences, Austria

In this overview talk we give a broad reflection of frame theory and the connection to various application areas, in particular acoustics.

We will introduce the basic definition of frames and motivate it by coherent states. We will talk about time-frequency analysis and its link to frame theory. As a particular form of quantization operators we will present frame multipliers. We will show how they are applied in signal processing as time-variant filters. We will introduce the representation of operators using frames, and show the link to acoustical simulations. We will present sound signals of a particular application in acoustics, audio inpainting. Finally, we will hint at the connection of frames to deep neural networks, as an outlook.

C*-Algebraic Methods in Local Quasi-Hermitian Physics

Jacob Barnett*

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Quasi-Hermitian quantum theory defines a framework of quantum theory which defines new notions of locality. For example, in the tensor product model, introducing an entangled metric operator lifts a hidden implicit assumption of the model; namely, quasi-Hermitian tensor product models admit physical inner products which do not factorize with the tensor product.

This talk will discuss two applications of the theory of C^* -algebras to quasi-Hermitian quantum theory.

- Firstly, representation theory can generate nontrivial quasi-Hermitian models from simpler ones. Examples include the models of [1, 2].
- Secondly, I will characterize local observable algebras and expectation values. A key result is that Bell's inequality violations in quasi-Hermitian theories never exceed the Tsirelson bound set in Hermitian theories.

References

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- [2] Barnett, Jacob L., and Yogesh N. Joglekar. "Effects of detuning on *PT*-symmetric, tridiagonal, tight-binding models." arXiv preprint arXiv:2302.13204 (2023).

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Bounded elements and topologies on quasi *-algebras defined by means of sufficient families of sesquilinear forms

Giorgia Bellomonte Dipartimento di Matematica e Informatica Università degli Studi di Palermo

Starting from a pure algebraic setting, two notions of bounded element of a quasi *algebra $(\mathfrak{A}, \mathfrak{A}_0)$ possessing a sufficiently large family \mathcal{M} of invariant positive sesquilinear (i.p.s.) forms on $\mathfrak{A} \times \mathfrak{A}$ are introduced and studied (i.p.s. forms on $\mathfrak{A} \times \mathfrak{A}$ are interesting since they allow a GNS construction similar to that defined by a positive linear functional on a *-algebra \mathfrak{A}_0). In particular, the notion of bounded element with respect to the family \mathcal{M} and that one related to the order defined by \mathcal{M} are introduced without any hypothesis of continuity of the forms; if \mathcal{M} is a *sufficient* family of i.p.s forms on $\mathfrak{A} \times \mathfrak{A}$ (in the sense that, for every $a \in \mathfrak{A}, a \neq 0$ there exists $\varphi \in \mathcal{M}$ such that $\varphi(a, a) > 0$), the two notions turn out to be equivalent and every *-representation produces a bounded operator when acting on a bounded element. Moreover, some topologies generated by means of the forms of a sufficient family \mathcal{M} are introduced and studied. When the family \mathcal{M} is sufficient and some other hypotheses of regularity are fulfilled, the notion of locally convex quasi GA*-algebra can be introduced. It is characterized by the fact that its topology is equivalent to that generated by \mathcal{M} ; their main feature consists in the fact that the family of their bounded elements, with respect to the family \mathcal{M} , is a dense C*-algebra.

Quantum hybrids

Raffaele Carlone

Università Federico II Napoli, Italy

In the design of microscopic scale devices arises the need for models in which quantum dynamics occurs on manifolds of different dimensionality. While nanoscale circuits have given rise to quantum graph theory, lithographic techniques and more generally the fabrication of films with contacts requires that the wires are connected to planes and that a possibly nonlinear dynamic takes place. The development of these models has been started, thanks to the technique of point interactions, in several papers for the linear case. In this talk we want to present some recent results that generalize to the nonlinear case the hybrid models between plane and straight line with Schrödinger dynamics.

Hermitian and non-Hermitian interactions mediated by a photonic bath

Francesco Ciccarello¹ ¹NEST-Pisa and University of Palermo, Italy

It is well-known that a photonic environment can mediate effective 2nd-order interactions between quantum emitters described by a generally non-Hermitian effective Hamiltonian. Current technology nowadays allows to fabricate relatively sophisticated photonic lattices (e.g. in circuit QED) and to couple them locally to controllable quantum emitters. In particular, as a photonic bath can be tailored so as to possess non-trivial topological and/or non-Hermitian properties, it is natural to wonder whether it can mediate atomic interactions enjoying analogous features Here, I will discuss some general theorems linking the topology of the emitters to that of the photonic bath in terms of both Hermitian and non-Hermitian topological invariants. Remarkably, the atomic topology can be shown to be the same as or opposite to the photonic one, depending on the (non-)Hermiticity of the bath and the number of spatial dimensions. We in particular predict possible occurrence of atomic boundary modes with group velocity opposite to the photonic one.

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- [2] F. Roccati, M. Bello, Z. Gong, M. Ueda, F. Ciccarello, A. Chenu, A. Carollo, arXiv:2303.00762

Continuous harmonic functions in a ball that are not in $H^{\frac{1}{2}+\epsilon}$

R. Bramati¹, M. Dalla Riva², B. Luczak³

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The starting point is a nonlinear transmission problem that describes a composite material with a thermoactive interface. With G. Mishuris (Aberystwyth University, UK) we have demonstrated that, under natural assumptions, the nonlinear transmission problem has very weak solutions. These are solutions that lack the regularity required to solve the problem in the standard variational sense. To determine the optimality of the regularity obtained, we aimed to provide explicit examples. It turned out that constructing harmonic functions, which are continuous up to the boundary but not differentiable in the standard or variational sense, was the key step in preparing these examples. We took inspiration from Hadamard's classic paper (1906) and obtained continuous harmonic functions in a ball in \mathbf{R}^n that are not in $H^{\frac{1}{2}+\epsilon}$ for $\epsilon > 0$.

Revivals in the linear Schrödinger equation

George Farmakis

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When posed under periodic boundary conditions, the free linear Schrödinger equation exhibits the revival and fractalisation effects, at rational and irrational times respectively. At rational times, the solution is decomposed into a finite number of translated copies of the initial state (*revival*). Consequently, if the initial function has a jump discontinuity, then the solution also exhibits finitely many jump discontinuities. On the other hand, at irrational times the solution evolves to a continuous, but nowhere differentiable function (*fractalisation*). In this talk, we will discuss recent extensions of the revival effect to the case of the linear Schrödinger equation, with a complex potential, subject to zero Dirichlet boundary conditions. This is joint work with Lyonell Boulton (Heriot-Watt University, UK), Beatrice Pelloni (Heriot-Watt University, UK) and David Smith (Yale-NUS College & National University of Signapore, Signapore).

On modular frames in Hilbert C^* -modules

Michael Frank

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We give an introduction into modular frame theory of Hilbert C^* -modules focussing on key facts of the existing part of the theory. Special emphasis will be put on C^* -algebras of coefficients like matrix algebras, compact C^* -algebras, W^* -algebras or monotone complete C^* -algebras. As a limitation, also examples of Hilbert C^* -modules without modular frames will be characterized. Some possible modifications in the basic definition and some fields of application will be indicated.

Toda field theories and Calogero models associated to infinite Weyl groups

Andreas Fring

School of Science & Technology - City, University of London

Many theories can be formulated universally in terms of Lie algebraic roots. Examples are conformal field theories that can be expressed in terms of the simple roots of finite Lie algebras, massive field theories that can be written in terms of simple roots of the affine Kac-Moody algebras and Calogero (Moser-Sutherland) models that require the entire root system of the finite Lie algebras in their formulation. Here we discuss extensions to similar systems based on hyperbolic and Lorentzian Kac-Moody algebras. We discuss various properties of these models, including their integrability and invariance with regard to infinite Weyl groups of affine, hyperbolic and Lorentzian type.

Gelfand triplets, Lie algebras and Quantum Mechanics

Manuel Gadella

Universidad de Valladolid

Gelfand triplets are used in Quantum Mechanics with a variety of purposes, including a rigorous presentation of the Dirac Formulation of Quantum Mechanics, or a mathematical construction of certain structures linked to insatiable quantum states, such as Gamow vectors. In addition, Gelfand triplets permit the combination of a series of structures commonly used in quantum textbooks, such as discrete and continuous basis or other features. Here, we present a limited number of examples thereof. Gelfand triplets will serve as a support of representations in terms of continuous operators of some examples of Lie algebras.

Completely periodic pseudodifferential operators and existence of Gabor frames

Gianluca Garello¹, Alessandro Morando² ¹ Mathematics Department, University of Torino, Italy ² DICATAM, University of Brescia, Italy

A Gabor system $\mathcal{G}(g, a, b)$ is a sequence of the type $\{g_{hk} = e^{2\pi i bk \cdot x}g(x - ah)\}_{h,k \in \mathbb{Z}^d}$, with g measurable function on \mathbb{R}^d , a, b > 0.

 $\mathcal{G}(g, a, b)$ is said to be a frame in $L^2(\mathbf{R}^d)$ if $A ||f||_{L^2}^2 \leq \sum_{h,k \in \mathbf{Z}^d} |(f, g_{h,k})|^2 \leq B ||f||_{L^2}^2$, for some A, B > 0 and any $f \in L^2(\mathbf{R}^d)$. Gabor frames play an important role in signal processes.

A wide literature, see for example [2], [3], [4], [5], is devoted in finding conditions on the window g and the lattice parameters a, b > 0, which allow the corresponding Gabor system to be a frame in $L^2(\mathbf{R}^d)$, so that the Gabor operator $S_{g,g}f = \sum_{h,k\in\mathbf{Z}^d} (f,g_{hk})g_{hk}$ is invertible in $\mathcal{L}(L^2)$ and a reconstruction formula $f = \sum_{h,k\in\mathbf{Z}^d} (f,g_{hk})\gamma_{h,h}$ is available, with $\gamma = S_{g,g}^{-1}g$. To this respect the very basic assumption is that a and b are "small enough".

In this talk we show sufficient conditions which allow $\mathcal{G}(g, a, b)$ to be a frame, by using the results of continuity and invertibility in $L^p(\mathbf{R}^d)$ for pseudodifferential operators with symbols $\sigma(x,\xi)$ periodic in both the variable, introduced in [1].

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- [2] K. Gröchenig, The mystery of Gabor frames, J. Fourier Anal. Appl., 20 n. 4 (2014), 865-895.
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- [4] D.F. Walnut, Continuity Properties of the Gabor Frame Operator, J. Math. Anal. Appl. 165, (1992), 479-504.
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Quantum circuit complexity for light polarisation and spin.

Jean-Pierre Gazeau

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Inspired by Nielsen's geometric approach to computation [1, 2], we present a type of quantum circuit complexity for open systems on a general level through two elementary examples, namely the quantum orientations in the plane, or, equivalently the linear polarisations of the light, and spin 1/2 (or two level) quantum systems. Precisely we examine the evolution of mixed quantum states submitted to interact with a sequence of gates, for instance polarizers in the case of the light. The fact that the system is open means we examine a sequence of N density matrices, each one evolving along a Lindbladt equation [4] during the time interval between two gates. The real n-dimensional case is also examined. The presentation is based on the work [3] lying in the continuation of the previous [5, 6, 7].

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Distributions for nonsymmetric weakly monotone position operators and weakly monotone C^* -algebra

Maria Elena Griseta

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We investigate the vacuum distribution, under an appropriate scaling, of a family of partial sums of nonsymmetric position operators on the weakly monotone Fock spaces $(\mathfrak{F}_{WM}(\mathcal{H}))$ on the Hilbert space \mathcal{H} , and we give an abstract representation for the weakly monotone C^* -algebra, i.e. the C^* -algebra generated by finitely many annihilation operators acting on $\mathfrak{F}_{WM}(\mathcal{H})$.

We first show that any single operator has the vacuum law belonging to the free Meixner class. After establishing some relations between the combinatorics of Motzkin and Riordan paths, we give a recursive formula for the vacuum moments of the law of any finite sum. We also investigate the asymptotic measure for these sums, which turns out to belong to the free Meixner class, with an atomic and an absolutely continuous part, both explicitly computed.

Successively, we show that the weakly monotone C^* -algebra is isomorphic to a suitable quotient of a Cuntz-Krieger C^* -algebra \mathcal{O}_A corresponding to a suitable matrix A. Finally, we prove that the diagonal subalgebra of the weakly monotone C^* -algebra is a MASA. This is joint work with V. Crismale and J. Wysoczański.

Multi-Dimensional Quantum Walks: a Playground of Dirac and Schrödinger Particles

Manami Yamagishi¹, Naomichi Hatano² (speaker), Ken-Ichiro Imura², Hideaki Obuse³

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We propose a new multi-dimensional discrete-time quantum walk (DTQW), whose continuum limit is an extended multi-dimensional Dirac equation, which can be further mapped to the Schrödinger equation. We show in two ways that our DTQW is an excellent measure to investigate the two-dimensional (2D) extended Dirac Hamiltonian and higher-order topological materials. First, we show that the dynamics of our DTQW resembles that of a 2D Schrödinger harmonic oscillator; see Fig. 1. Second, we find in our DTQW topological features of the extended Dirac system: see Fig. 2. By manipulating the coin operators, we can generate not only standard edge states but also corner states.



Figure 1: The expectation values $\langle x \rangle$ and $\langle y \rangle$ of the position of 2D DTQW for (a) $0 \le T \le$ 500 and for (b) $500 \le T \le 1000$. Black circles indicate the values at the beginning of time evolution; they turn into orange as time goes on. We set $\hbar = a = \Delta t = 1$.



Figure 2: Dispersion relation of the quasi-energy spectra $E(k_y)$ with $\theta_1 = -\theta_2 = \pi/3$. (a) $\theta_y = 0$ and (b) $\theta_y = \pi/50$ without randomness, and (c) $\theta_y = 0$ with randomness $\Delta \theta_{x0}(x) \in [-0.25, 0.25]$. The central part of the dispersion is enlarged in the upper right corner in each panel. We set $\hbar = a = \Delta t = 1$.

The present talk is based on Ref. [1]. The second author, Naomichi Hatano, will present the talk.

 Manami Yamagishi, Naomichi Hatano, Ken-Ichiro Imura, and Hideaki Obuse, Proposal of multidimensional quantum walks to explore Dirac and Schrödinger systems, Phys. Rev. A 107, 042206 (14 pp) (2023). [LINK]

Decomposition theory of unbounded observable algebras

Hiroshi Inoue

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M. Tomita defined the concepts of observables and observable algebras and studied them in order to bridge quantum mechanics and mathematics. Tomita's idea for this question is to consider the trio (A, x, y^*) consisting of an observable A, of two states x, y as an observable, which is called a trio observable, and to introduce an algebraic and topological structure in the set of trio observables. This means that an operator observable A having two different states can be regarded as two different observables. A. Inoue has introduced and developed this theory in [2]. The operator part of a Tomita's observable is always a bounded linear operator on \mathcal{H} , however an operator observable in quantum mechanics is unbounded, and the GNS-representation of a positive linear functional on a *-algebra is unbounded. This is our motivation for defining and studying unbounded observable algebras, which are an unbounded generalization of Tomita's observable algebras in [3]. Here we state this roughly. Let \mathcal{D} be a dense subspace in a Hilbert space \mathcal{H} and write $\mathcal{D}^* = \{\xi^* \in \mathcal{H}^*; \xi \in \mathcal{D}\}$, where ξ^* is an element of the dual space \mathcal{H}^* of \mathcal{H} defined by $\langle \xi^*, x \rangle := (x|\xi)$ for all $x \in \mathcal{H}$. We denote by $\mathcal{L}^{\dagger}(\mathcal{D})$ the set of all linear operators X from \mathcal{D} to \mathcal{D} satisfying $D(X^*) \supset \mathcal{D}$ and $X^*\mathcal{D} \subset \mathcal{D}$, where X^* is the adjoint of X and $D(X^*)$ is the domain of X^* . Then $\mathcal{L}^{\dagger}(\mathcal{D})$ is a *-algebra consisting of closable operators in \mathcal{H} equipped with the usual operations $(X+Y,\alpha X \text{ and } XY)$ and the involution $X \mapsto X^{\dagger} := X^* \lceil_{\mathcal{D}}$ (the restriction of X^* to \mathcal{D}). A quadruplet $A = (A_0, \xi, \eta^*, \mu)$ of $A_0 \in \mathcal{L}^{\dagger}(\mathcal{D}), \xi, \eta \in \mathcal{D}$ and $\mu \in \mathbb{C}$ is called an (unbounded) quadruplet observable on \mathcal{D} and denoted by $Q^{\dagger}(\mathcal{D})$ of all quadruplet observables on \mathcal{D} . Referring observables, states and expectations in the standard Hilbert space formulation of the quantum mechanics, we define the algebraic operations and involution \dagger on $Q^{\dagger}(\mathcal{D})$ as follows:

$$\begin{aligned} A + B &= (A_0 + B_0, \xi + \zeta, \eta^* + \chi^*, \gamma + \sigma), \\ \alpha A &= (\alpha A_0, \alpha \xi, \alpha \eta^*, \alpha \gamma), \\ AB &= (A_0 B_0, A_0 \zeta, (B_0^{\dagger} \eta)^*, (\zeta | \eta)), \\ A^{\dagger} &= (A_0^{\dagger}, \eta, \xi^*, \bar{\gamma}) \end{aligned}$$

for $A = (A_0, \xi, \eta^*, \gamma), B = (B_0, \zeta, \chi^*, \sigma) \in Q^{\dagger}(\mathcal{D})$ and $\alpha \in \mathbb{C}$. A *-subalgebra of the *algebra $Q^{\dagger}(\mathcal{D})$ is said to be a Q^{\dagger} -algebra on \mathcal{D} . Investigating Q^{\dagger} -algebras, we may deal with various physical phenomenons. For an element $A = (A_0, \xi, \eta^*, \gamma)$ of a Q^{\dagger} -algebra \mathfrak{A} on \mathcal{D} we write $\pi(A) = A_0$, $\lambda(A) = \xi$, $\lambda^*(A) = \eta^*$ and $\mu(A) = \gamma$. Then π is a (possibly unbounded) *-representation of \mathfrak{A} on \mathcal{D} (namely, a *-homomorphism of \mathfrak{A} into $\mathcal{L}^{\dagger}(\mathcal{D})$), λ is a vector representation of \mathfrak{A} into \mathcal{D} (namely, a linear mapping of \mathfrak{A} into \mathcal{D} satisfying $\lambda(AB) = \pi(A)\lambda(B)$ for all $A, B \in \mathfrak{A}$, λ^* is a vector representation of \mathfrak{A} into \mathcal{D}^* and μ is a positive linear functional on \mathfrak{A} . A trio $(\pi(A), \lambda(A), \lambda^*(A))$ obtained by cutting a quadruplet observable A on \mathcal{D} is called a trio observable on \mathcal{D} , and the set $T^{\dagger}(\mathcal{D})$ of all trio observables on \mathcal{D} is also a *-algebra without identity under operations and the involution [†] as those in the case of $Q^{\dagger}(\mathcal{D})$. A *-subalgebra of $T^{\dagger}(\mathcal{D})$ is called a T^{\dagger} -algebra on \mathcal{D} . When $\mathcal{D} = \mathcal{H}$, a quadruplet (resp. trio) observable A is a Tomita's quadruplet (resp. trio) observable, namely, $\pi(A)$ is a bounded linear operator on \mathcal{H} , $\lambda(A) \in \mathcal{H}$ and $\lambda^*(A) \in \mathcal{H}^*$. The set $Q^*(\mathcal{H})$ (resp. $T^*(\mathcal{H})$) of all Tomita's quadruplet (resp. trio) observables on \mathcal{H} is a Banach *-algebra without identity equipped with the above operations A + B, αA , AB, the involution $A^{\sharp} := (\pi(A)^*, \lambda^*(A)^*, \lambda(A)^*, \mu(A))$ (resp. $A^{\sharp} = (\pi(A)^*, \lambda^*(A)^*, \lambda(A)^*)$) and the norm $||A|| := \max(||\pi(A)||, ||\lambda(A)||, ||\lambda^*(A)||, |\mu(A)|)$ (resp. $||A|| := \max(||\pi(A)||, ||\lambda(A)||, ||\lambda(A)||$ $\|\lambda^*(A)\|)$.

norm $||A|| := \max(||\pi(A)||, ||\lambda(A)||, ||\lambda^*(A)||, ||\mu(A)|)$ (resp. $||A|| := \max(||\pi(A)||, ||\lambda(A)||, ||\lambda^*(A)||)$).

A *-subalgebra of $Q^*(\mathcal{H})$ (resp. $T^*(\mathcal{H})$) is called a Q^* -algebra (resp. a T^* -algebra) on \mathcal{H} , and a closed *-subalgebra of the Banach *-algebra $Q^*(\mathcal{H})$ (resp. $T^*(\mathcal{H})$) is called a CQ^* -algebra (resp. CT^* -algebra) on \mathcal{H} . Here we remark that in [2] the involution on $Q^*(\mathcal{H})$ and $T^*(\mathcal{H})$ is denoted by $A \to A^{\sharp}$ as we did, however we denote the involution on $Q^{\dagger}(\mathcal{D})$ and $T^{\dagger}(\mathcal{D})$ by $A \to A^{\dagger}$.

In this talk, we tried to build the basic theory of unbounded Tomita's observable algebras called T^{\dagger} -algebras which are related to unbounded operator algebras, especially unbounded Tomita-Takesaki theory [1], operator algebras on Krein spaces [5], studies of positive linear functionals on *-algebras and so on. And we defined the notions of regularity, semisimplicity and singularity of T^{\dagger} -algebras and characterized them. We shall proceed further studies of T^{\dagger} -algebras and investigate whether a T^{\dagger} -algebra is decomposable into a regular part and a singular part [4].

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Semi-Fredholm theory in unital C*-algebras

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Kečkić and Lazović introduced in [4] an axiomatic approach to Fredholm theory by considering Fredholm type elements in a unital C^* -algebra as a generalization of C^* - Fredholm operators on the standard Hilbert C^* -module introduced by Mishchenko and Fomenko in [5], and of Fredholm operators on a properly infinite von Neumann algebra introduced by Breuer in [1, 2]. In this presentation we establish semi-Fredholm theory in unital C^* algebras as a continuation of the approach by Kečkić and Lazović. We introduce the notion of semi-Fredholm type elements and semi-Weyl type elements. We prove that the difference of the set of semi-Fredholm elements and the set of semi-Weyl elements is open in the norm topology, that the set of semi-Weyl elements is invariant under perturbations by finite type elements, and several other results generalizing their classical counterparts. Also, we illustrate applications of our results to the special case of properly infinite von Neumann algebras and we obtain a generalization of the punctured neighbourhood theorem in this setting.

This talk is based on [3].

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Breaking quantum speed limit and generating super-quantum correlations with non-self-adjoint Hamiltonians

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Traditional quantum theory is based on a Dirac-self-adjoint generator of time-translation, i.e. a Hermitian Hamiltonian H. In addition to norm conservation, such dynamics constrain the time it takes for a state to evolve into an orthogonal state ("orthogonality" is with respect to the standard inner product) and consequently, temporal correlations between multi-time measurements of an observable [1]. For a two-level system, (qubit) the said time τ_{QSL} is bounded below by the minimum of the Mandelstam-Tamm bound $\tau_{\text{MT}} = \pi/2\Delta H$ and the Margolis-Levitin bound $\tau_{\text{MT}} = \pi/2\langle H \rangle$ [2]. This quantum speed limit caps the three-time correlations, called the Leggett-Garg parameter to $-3 \leq K_3 \leq 3/2$ [3], with values below 1 indicating classical behavior. For a dissipative system described by linear quantum maps (Lindblad equation), these results remain valid, thereby providing nearuniversal constraints on the quantum speed limit τ_{OSL} and temporal correlations K_3 [4].

I will show that these limits can be broken in quantum systems governed by non-self-adjoint Hamiltonians. Such dynamics occur in post-selected, minimal open quantum systems, where trace-preservation constraint leads to a nonlinear equation of motion. I will present theoretical results showing that the temporal correlation values K_3 can exceed 1.5 and evolution times can be shorter than the unified quantum speed limit value τ_{SQL} . I will then present results from experiments on a single, trapped ion — a minimal quantum bit — that demonstrate these super-quantum correlations. Our results show that non-self-adjoint Hamiltonians generate stronger temporal correlations faster [5].

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^{*}Work done in collaboration with Sourin Das group (IISER, Kolkata) and David Allcock group (University of Oregon, Eugene)

Is the optimal rectangle a square?

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We give a light talk on an optimality of a square in geometry and physics. First, we recollect classical geometric results that the square has the largest area (respectively, the smallest perimeter) among all rectangles of a given perimeter (respectively, area). Second, we recall that the square drum has the lowest fundamental tone among all rectangular drums of a given area or perimeter and reinterpret the result in a quantum-mechanical language of nanostructures. As the main body of the talk, we present our recent attempts to prove the same spectral-geometric properties in relativistic quantum mechanics, where the mathematical model is a matrix-differential (Dirac) operator with complex (infinite-mass) boundary conditions. It is frustrating that such an illusively simple and expected result remains unproved and apparently out of the reach of current mathematical tools.

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Hamiltonians generated by the Parseval frames

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The purpose of this talk is to analyze Hamiltonians generated by Parseval frames. The interest in such operators stems from their physical significance, as the mathematical Hilbert space in which a physical system is defined may include states that cannot be occupied by the system itself. By exploring the transition from orthonormal bases to Parseval frames, we can better understand the spectrum changes that are possible. Our analysis primarily relies on the Naimark dilation theorem for Parseval frames.

The results presented in this talk are the outcome of the joint work with Fabio Bagarello [1, 2].

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On the existence of the fundamental transfer matrix in two-dimensional potential scattering and the propagating-wave approximation

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Potential scattering admits a dynamical formulation in terms of a fundamental notion of transfer matrix. This is a linear operator that is given by the evolution operator for an effective non-self-adjoint Hamiltonian operator [1, 2]. This approach to potential scattering has so far led to a number of interesting developments in two and three dimensions. The most notable of these are the construction of the first examples of short-range potentials for which the first Born approximation is exact [3], potentials that display broadband omnidirectional or unidirectional invisibility [3, 4], and a singularity-free treatment of deltafunction point scatterers lying on a line in two dimensions and on a plane in three dimensions [5, 6, 7]. This talk presents a first step towards a rigorous proof of the existence of the fundamental transfer matrix in two dimensions. It offers a solution of this problem within the context of propagating-wave approximation [8]. This is an approximation scheme that ignores the contribution of the evanescent waves to the scattering amplitude and is valid for high energies and weak potentials [9].

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Mean and variance of the cardinality of particles in polyanalytic Ginibre processes via a quantization method

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We discuss the mean and variance of the number "point-particles" \sharp_{D_R} inside a disk D_R centered at the origin of the complex plane \mathbb{C} and of radius R > 0 with respect to a Ginibre-type (polyanalytic) process of index $m \in \mathbb{Z}_+$ by quantizing the phase space \mathbb{C} via a set of generalized coherent states $|z, m\rangle$ of the harmonic oscillator on $L^2(\mathbb{R})$. By this procedure, the spectrum of the quantum observable representing the indicator function χ_{D_R} of D_R (viewed as a classical observable) allows to compute the mean value of \sharp_{D_R} . The variance of \sharp_{D_R} is obtained as a special eigenvalue of a quantum observable involving to the auto-convolution of χ_{D_R} . By adopting a coherent states quantization approach, we seek to identify classical observables on \mathbb{C} , whose quantum counterparts may encode the first cumulants of \sharp_{D_R} through spectral properties.

This is a joint work with Mohamed Mahboubi and Othmane El Moize.

On regularity for double phase integral functionals

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Double phase problems recently received a large interest in the mathematical literature due to their applications to anisotropic variational energies in integral form and there are still many open questions [1, 2]. In this context, we study regularity properties for minimizers of a class of double phase integrals characterized by nonstandard growth conditions. The main novelty is that the underlying energy features a non-uniformly elliptic integrand involving different growth conditions and a lack of homogeneity. We develop a few intrinsic methods in order to prove local and global higher integrability properties. The approach is purely variational. The setting is the general context of metric measure spaces, endowed with a doubling metric measure and supporting a Poincaré inequality.

This is a joint project work with Cintia Pacchiano Camacho (University of Calgary) and Juha Kinnunen (Aalto University) [3, 4].

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Mean-dispersion principles and the Wigner transform

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In this talk we present uncertainty principles for families of orthonormal functions in $L^2(\mathbb{R})$ in connection with time-frequency analysis. When talking about uncertainty principles, in harmonic analysis, one refers to a class of theorems giving limitations on how much a function and its Fourier transform can be both localized at the same time. Different meanings of the word "localized" give rise to different uncertainty principles. There are, moreover, uncertainty principles giving not only limitations on the localization of a single function and its Fourier transform, but on how such limitations behave, becoming stronger and stronger, when adding more and more elements of an orthonormal system in L^2 . In this paper we focus in particular on results of this type involving means and variances. For $f \in L^2(\mathbb{R})$ we define the associated mean and the associated variance by

$$\mu(f) := \frac{1}{\|f\|^2} \int_{\mathbb{R}} t |f(t)|^2 dt \quad \text{and} \quad \Delta^2(f) := \frac{1}{\|f\|^2} \int_{\mathbb{R}} |t - \mu(f)|^2 |f(t)|^2 dt,$$

respectively. The dispersion associated with f is $\Delta(f) := \sqrt{\Delta^2(f)}$. An uncertainty principle for orthonormal sequences, that constitutes the starting point of the present work, is due to Shapiro.

Theorem (Shapiro's Mean-Dispersion Principle). There does not exist an infinite orthonormal sequence $\{f_k\}_{k\in\mathbb{N}_0}$ in $L^2(\mathbb{R})$ such that all $\mu(f_k)$, $\mu(\hat{f}_k)$, $\Delta(f_k)$, $\Delta(\hat{f}_k)$ are uniformly bounded.

Some refinements of this result have been obtained in the literature, and a quantitative version of it has been proved by Jaming and Powell in [1].

In this talk we present uncertainty principles of mean-dispersion type involving quadratic time-frequency representations (in particular the Wigner transform) applied to the elements of an orthonormal system in $L^2(\mathbb{R})$. Such results are given in a quantitative form, including in particular the results of [1] and the classical Shapiro's mean-dispersion principle. In particular, if $\{f_k\}_{k\in\mathbb{N}_0}$ is an orthonormal sequence in $L^2(\mathbb{R})$, we prove that for every $n \ge 0$

$$\sum_{k=0}^{n} \int_{\mathbb{R}^{2}} (x^{2} + \xi^{2}) |W(f_{k})(x,\xi)|^{2} dx d\xi \geq \frac{(n+1)^{2}}{2}.$$

As a consequence, we show that there does not exist an infinite orthonormal sequence $\{f_k\}_{k\in\mathbb{N}_0}$ in $L^2(\mathbb{R})$ satisfying $\mu(f_k) = \mu(\hat{f}_k) = 0$ such that all the traces of the covariance matrices of $|W(f_k)(x,\xi)|^2$ are uniformly bounded.

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Pseudo-hermitian Hamiltonians at finite temperature

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We analyze the thermodynamical properties of two different pseudo-hermitian models, the Swanson model [1] and a pseudo-hermitian extension of the Da Providencia-Schütte model [2].

The dynamical properties of the PT-symmetric Swanson Model are studied at finite temperature, both in the PT-unbroken and in the PT-broken symmetry phase. We analyse the behaviour of the system, which is initially at equilibrium at a given temperature, when it is perturbed by a periodic interaction. We adopt The Double Green Function Formalism to describe the thermodynamics of this quantum the system. This formalism have been extensively used to study the time evolution under the action of a given self-adjoint Hamiltonian. In this work, we extend The Double Green Function Formalism to include pseudo-hermitian Hamiltonians.

As an another example of the role of pseudo-hermicity in quantum thermodynamics, we study a hybrid system based on the Da Providencia-Schütte Hamiltonian. The model consists of bosons, i.e. photons in a cavity, interacting with an ensemble of spins through a pseudo-hermitian hamiltonian. We compute the exact partition function of the system, and from it, we derive the statistical properties of the system. Finally, we evaluate the work that can be extracted from the system by performing an Otto cycle and discuss the advantages of the proposed pseudo-hermitian interaction. Also the behaviour of the system is analyzed in the vecinity of Exceptional Points.

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Hilbert space representations of the meromorphic Weyl algebra

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The meromorphic Weyl algebra is the unital complex *-algebra with hermitean generator u, v and defining relation

$$uv - vu = iv^2$$
.

(It is obtained from the "usual" Weyl algebra with hermitean generators q, p and relation pq - qp = -i1 by setting formally $u = p, v = q^{-1}$.)

The talk deals with operator representations of the meromorphic Weyl algebra on Hilbert space. A counter-part of the Weyl relation for the canonical commutation relation is formulated and a classification theorem of the corresponding well-behaved representations is obtained. The C^* -algebra associated to the well-behaved representations is considered.

Many-particle systems with contact interactions in dimension three

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Hamiltonians with contact (or zero-range) interactions are useful models to analyze the behaviour of quantum systems at low energy in different contexts. In this talk we discuss the mathematical aspects of the construction of such Hamiltonians in dimension three as self-adjoint and lower bounded operators in the appropriate Hilbert space. We first consider the case of a system made of three identical bosons. In order to avoid the fall to the center phenomenon emerging in the standard Ter-Martirosyan Skornyakov (TMS) Hamiltonian, known as Thomas effect, we develop in detail a suggestion given in a seminal paper of Minlos and Faddeev in 1962 and we construct a regularized version of the TMS Hamiltonian. The regularization is given by an effective three-body force, acting only at short distance, that reduces to zero the strength of the interactions when the positions of the three particles coincide. The construction is then extended to the case of an arbitrary number of interacting bosons.

Sampling type operators for the study of eye fundus images

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The talk is motivated by the theory of sampling-type operators. Among them, the sampling Kantorovich operator represents a useful tool for dealing with not necessarily continuous functions [1]. Its muldimensional version [5] has been implemented and allows not only to reconstruct, but also to increase the information content of images, as it acts both as a low-pass filter and as a rescaling algorithm [3]. Indeed, Sampling Kantorovich algorithm has been used, with satisfactory results, in e.g., [2, 4, 6].

The talk is focused on some recent results which constist in the study on eye fundus images of healthy and diabetic patients. Taking benefits from its reconstruction and enhancing properties, the sampling Kantorovich algorithm is used to process the considered images, after registration and averaging processes. Moreover, a hybrid segmentation procedure applied on superficial capillary plexus images (SCP) and one using the local Phansalkar method on choriocapillary images (CC) are exploited in order to asses a cluster counting process which is based on finding connected regions according to the 8–adjacency criterion. The results achieved on the healthy and diabetic patients show that the novel strategy allows to obtain accurate data from both a mathematical and a clinical point of view.

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Pseudospectra of generalized Schrödinger operators

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Let A be a bounded linear operator on $\ell^p(\mathbb{Z})$. The minimum of

$$\nu(A) := \inf\{\|Ax\| : \|x\| = 1\}$$
(1)

and $\nu(A^*)$ is the inverse of the norm, $||A^{-1}||$, of the inverse of A. We approximate $\nu(A)$ by restricting the infimum (1) to elements $x \in \ell^p(\mathbb{Z})$ with support of diameter D, and we quantify the approximation error in terms of D.

For the simple case of one-sided as well as bi-infinite matrices,

$$H(b) = \begin{pmatrix} \ddots & \ddots & & & & \\ \ddots & * & \bullet & & & \\ \ddots & b_{-1} & * & \bullet & & \\ & + & b_0 & * & \bullet & \\ & & + & b_1 & * & \ddots \\ & & & \ddots & \ddots & \ddots \end{pmatrix}$$

with one varying diagonal (no matter which one) and finitely many constant diagonals (here simplified as +, * and \bullet), considered as acting on $\ell^p(\mathbb{N})$, resp. $\ell^p(\mathbb{Z})$, we derive pseudospectral inclusions and Hausdorff-approximations, avoiding spectral pollution, purely by looking at the set of finite subwords of the potential b.

The talk is based on joint work with Fabian Gabel, Dennis Gallaun, Julian Grossmann, and Marko Lindner.

Mathematical models based on the approximation of discrete operators for the processing of thermographic images

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During this talk, I will discuss some mathematical models for the reconstruction and the processing of digital images, focusing in particular on one recently introduced, based on discrete operators of sampling type and successfully applied in various fields. I will examine it by dealing in particular with the processing of thermographic images for the study of the seismic vulnerability of buildings.