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DICAM

ON MULTIPLE SUPPORT RANDOM EXCITATION OF BRIDGE STRUCTURES

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Abstract.*Multiple support excitations of elastic multi-span beams are studied. Based on the common set of equations of motion an efficient formulation is developed in order to reduce the degrees of freedom. The resulting equations are formally identical to those that are valid for structures under uniform support excitations. Stationary random multiple support excitation is studied by an approximate Pseudo Excitation Method calculating the power spectral density matrix of the structural response vector.*

1 INTRODUCTION

Structures supported on several foundationssuch as bridges behave very complex when subjected to ground motions, e.g. earthquakes. Analysis of seismic response cannot be based on the single assumption that free-field ground motions are spatially uniform. Therefore common discretization procedures, originally derived for structures under uniform support excitations, must be extended accordingly resulting in a larger system of equations of motion, see e.g. [1] and [2].

The dynamic response of bridges subjected to deterministic multiple support excitation has been investigated by various researchers, [3], [4], [5]. Random vibrations of bridges have been analyzed generally by spectral analysis approach in the last two decades. In [6] the response of continuous two- and three-span beams to varying ground motions is evaluated and the validity of the commonly used assumption of equal support motion is examined. An extensive comparison of random vibration methods for multiple support seismic excitation analysis of long-span bridges can be found in [7].

In this paperanadvanced formulation for linear elastic multi-span beams under multiple support excitation is proposed in order to reduce the degrees of freedom in a mechanically consistent manner. The resulting differential equations are formally identical to those of structures under uniform support excitations. Applying the classical modal analysis approach, it becomes necessary to introduce time-dependent participation factors.

For stationary random multiple support excitation the Pseudo Excitation Method [8] is introduced, which includes the main effects of wave passage and site response.

2 GOVERNING EQUATIONS OF MOTION

The equation of motion of a discretized linear elastic beam subjected to *uniform* support excitation,

$$w_{g1}(t)=w_{g1}(t)=...=w_{gM}(t)=w_g(t), \tag{1}$$

reads, compare [1],

$$\mathbf{m}\ddot{\mathbf{u}}+\mathbf{c}\dot{\mathbf{u}}+\mathbf{k}\,\mathbf{u}=-\mathbf{m}\mathbf{e}^{\mathbf{s}}\,\ddot{w}_g \tag{2}$$

where $\mathbf{m}$, $\mathbf{c}$, $\mathbf{k}$ stand for the mass, damping, and stiffness matrix, respectively. $\mathbf{u}(t)$ denotes the vector of the nodal transverse deflections $w_i(t),\ i=1,...,N$. If the dicretization is extended to include also nodal rotations, $\mathbf{u}(t)$ contains additional rotatory degrees of freedom, and the corresponding system matrices have to be extended accordingly. The uniform ground acceleration is expressed by $\ddot{w}_{\mathbf{g}}(t)$. The influence vector $\mathbf{e}^{\mathbf{s}}$ represents the displacements (and rotations) of the masses resulting from static application of a ground displacement. In case of a lumped-mass model, where only nodal deflections (and no rotations) are considered, it is a vector with each element equal to unity, $\mathbf{e}^{\mathbf{s}}=\mathbf{1}$.

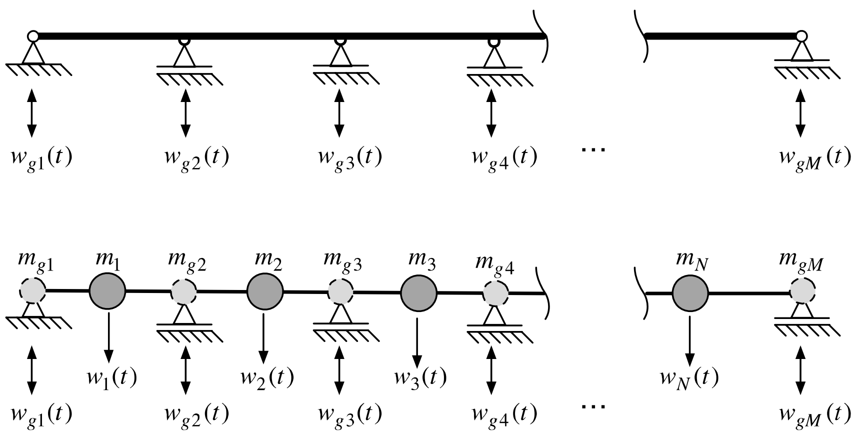

Figure 1: Multi-span beam and its discretization as lumped mass model.

Contrary, the coupled equations of motion of multi-span beams under *multiple* support excitation can be written formally as, compare [2],

$$\begin{bmatrix} \mathbf{m} & \mathbf{m_g} \\ \mathbf{m_g}^T & \mathbf{m_{gg}} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}^t \\ \ddot{\mathbf{u}}_g \end{bmatrix} + \begin{bmatrix} \mathbf{c} & \mathbf{c_g} \\ \mathbf{c_g}^T & \mathbf{c_{gg}} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}}^t \\ \dot{\mathbf{u}}_g \end{bmatrix} + \begin{bmatrix} \mathbf{k} & \mathbf{k_g} \\ \mathbf{k_g}^T & \mathbf{k_{gg}} \end{bmatrix} \begin{bmatrix} \mathbf{u}^t \\ \mathbf{u_g} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{p_g} \end{bmatrix}. \tag{3}$$

The displacement vector now contains two parts:

(a) $\mathbf{u^t}(t)$ includes the degrees of freedom of the beam, and

(b) $\mathbf{u_g}(t)$ contains the components of support excitation.

$\mathbf{m_g}, \mathbf{m_{gg}}, \mathbf{c_g}, \mathbf{c_{gg}}$, and $\mathbf{k_g}, \mathbf{k_{gg}}$ are submatrices associated with the support motion, and $\mathbf{p_g}(t)$ is the vector of support forces.

In the following a new, efficient representation of Eq. (3) is derived, which is related to the form of Eq. (2). Thus it becomes possible to use formally numerical procedures that are common in the field of structures under uniform support excitation.

3 MODELLING PROCEDURE

In a first step the individually prescribed support displacements, $w_{gj}(t),\ j = 1,..., M$, are interpreted as additional degrees of freedom, i.e., $u_k(t),\ k = (N+1),..., (N+M)$, see Fig. 2.

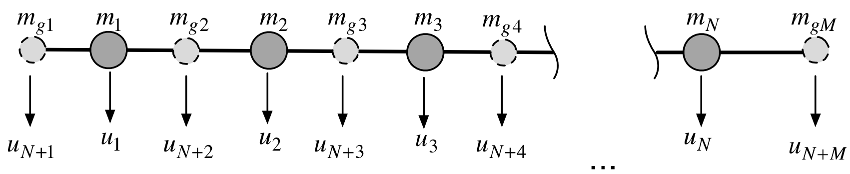

Figure 2: Free body diagram of the lumped mass model.

Next, the (singular) stiffness matrix of the complete discretized beam has to be evaluated, e.g., using the direct stiffness method by applying static unit deformations, which leads to

$$\mathbf{K} = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1N} & k_{1(N+1)} & \cdots & k_{1(N+M)} \\ k_{21} & k_{22} & \cdots & k_{2N} & \vdots & & \vdots \\ \vdots & & \ddots & & & & \\ k_{N1} & \cdots & & k_{NN} & k_{N(N+1)} & \cdots & k_{N(N+M)} \\ k_{(N+1)1} & \cdots & & k_{(N+1)N} & k_{(N+1)(N+1)} & \cdots & k_{(N+1)(N+M)} \\ \vdots & & & \vdots & \vdots & \ddots & \vdots \\ k_{(N+M)1} & \cdots & & k_{(N+M)N} & k_{(N+M)(N+1)} & \cdots & k_{(N+M)(N+M)} \end{bmatrix}. \tag{4}$$

Mass and damping matrices of Eq. (3) are of analogous form.
In the analysis of such dynamic system it is common to decompose the response into pseudo-static and dynamic response,

$$\mathbf{U}(t)=\begin{bmatrix}\mathbf{u}^{\mathbf{t}}(t)\\ \mathbf{u}_{\mathbf{g}}(t)\end{bmatrix}=\begin{bmatrix}\mathbf{u}^{\mathbf{s}}(t)\\ \mathbf{u}_{\mathbf{g}}(t)\end{bmatrix}+\begin{bmatrix}\mathbf{u}(t)\\ \mathbf{0}\end{bmatrix}. \tag{5}$$

The pseudo-static component satisfies the equation

$$\begin{bmatrix}\mathbf{k} & \mathbf{k}_{\mathbf{g}}\\ \mathbf{k}_{\mathbf{g}}^{T} & \mathbf{k}_{\mathbf{gg}}\end{bmatrix}\begin{bmatrix}\mathbf{u}^{\mathbf{s}}(t)\\ \mathbf{u}_{\mathbf{g}}(t)\end{bmatrix}=\begin{bmatrix}\mathbf{0}\\ \mathbf{p}_{\mathbf{g}}^{\mathbf{s}}(t)\end{bmatrix}, \tag{6}$$

from which one can solve for $\mathbf{u}^{\mathbf{s}}(t)$:

$$\mathbf{u}^{\mathbf{s}}(t)=-\mathbf{k}^{-1}\mathbf{k}_{\mathbf{g}}\mathbf{u}_{\mathbf{g}}. \tag{7}$$

Substituting Eqs. (5) and (7) into Eq. (3) results in

$$\mathbf{m}\ddot{\mathbf{u}}+\mathbf{c}\dot{\mathbf{u}}+\mathbf{k}\mathbf{u}=-\left[\mathbf{m}(-\mathbf{k}^{-1}\mathbf{k}_{\mathbf{g}})+\mathbf{m}_{\mathbf{g}}\right]\ddot{\mathbf{u}}_{\mathbf{g}}-\left[\mathbf{c}(-\mathbf{k}^{-1}\mathbf{k}_{\mathbf{g}})+\mathbf{c}_{\mathbf{g}}\right]\dot{\mathbf{u}}_{\mathbf{g}}\equiv\mathbf{p}_{eff}. \tag{8}$$

The vector of support forces can be expressed as

$$\begin{aligned}\mathbf{p}_{\mathbf{g}}(t)&=(-\mathbf{k}^{-1}\mathbf{k}_{\mathbf{g}}\mathbf{m}_{\mathbf{g}}^{T}+\mathbf{m}_{\mathbf{gg}})\ddot{\mathbf{u}}_{\mathbf{g}}+(-\mathbf{k}^{-1}\mathbf{k}_{\mathbf{g}}\mathbf{c}_{\mathbf{g}}^{\mathbf{T}}+\mathbf{c}_{\mathbf{gg}})\dot{\mathbf{u}}_{\mathbf{g}}\\ &+(-\mathbf{k}^{-1}\mathbf{k}_{\mathbf{g}}\mathbf{k}_{\mathbf{g}}^{T}+\mathbf{k}_{\mathbf{gg}})\mathbf{u}_{\mathbf{g}}+\mathbf{m}_{\mathbf{g}}^{T}\ddot{\mathbf{u}}+\mathbf{c}_{\mathbf{g}}^{\mathbf{T}}\dot{\mathbf{u}}+\mathbf{k}_{\mathbf{g}}^{T}\mathbf{u}\quad .\end{aligned} \tag{9}$$

Considering that either the damping terms in the effective forcing function $\mathbf{p}_{\mathbf{eff}}$ can be neglected when the motions are not uniform, or,

$$\mathbf{c}=a_1\mathbf{k},\quad \mathbf{c}_{\mathbf{g}}=a_1\mathbf{k}_{\mathbf{g}}, \tag{10}$$

or the damping forces are assumed to be proportional to the relative velocity vector instead to the absolute velocity, i.e.,

$$\begin{bmatrix}\mathbf{c} & \mathbf{c}_{\mathbf{g}}\\ \mathbf{c}_{\mathbf{g}}^{T} & \mathbf{c}_{\mathbf{gg}}\end{bmatrix}\begin{bmatrix}\dot{\mathbf{u}}^{\mathbf{t}}\\ \dot{\mathbf{u}}_{\mathbf{g}}\end{bmatrix}\rightarrow\begin{bmatrix}\mathbf{c} & \mathbf{c}_{\mathbf{g}}\\ \mathbf{c}_{\mathbf{g}}^{T} & \mathbf{c}_{\mathbf{gg}}\end{bmatrix}\begin{bmatrix}\dot{\mathbf{u}}\\ \mathbf{0}\end{bmatrix}, \tag{11}$$

hhen Eq. (8) simplifies to

$$\mathbf{m}\ddot{\mathbf{u}}+\mathbf{c}\dot{\mathbf{u}}+\mathbf{k}\mathbf{u}=-\underbrace{\left[\mathbf{m}(-\mathbf{k}^{-1}\mathbf{k}_{\mathbf{g}})+\mathbf{m}_{\mathbf{g}}\right]}_{\mathbf{M}}\ddot{\mathbf{u}}_{\mathbf{g}}. \tag{12}$$

Note, that in case of a lumped-mass model, $\mathbf{m}_{\mathbf{g}}$ is a null matrix, which is assumed in following derivations of section 4.

4 MODAL EXPANSION TECHNIQUE

In the following Eq. (12) is transformed into a set of uncoupled equations of motions by modal superposition, that is assuming

$$\mathbf{u}(t) = \sum_{i=1}^{N} \vec{\phi}_i \, y_i(t), \tag{13}$$

where $\vec{\phi}_i$ represents the eigenvectors, and y_i stands for the generalized coordinates. Inserting Eq. (13) into Eq. (12), pre-multiplying by $\vec{\phi}_k^T$, and considering the orthogonality conditions as well as the assumptions of proportional modes for the damped structure, transforms Eq. (12) into

$$\ddot{y}_i + 2\zeta_i\omega_i\dot{y}_i + \omega_i^2 y_i = -\Gamma_i\ddot{\bar{u}}_i = -\Gamma_i\mathbf{B}_i\,\ddot{\mathbf{u}}_\mathbf{g} = -\Gamma_i\sum_{k=1}^{m}\beta_{ki}\,\ddot{u}_{gk}, \; , \; i = 1,2,...,N \; , \tag{14}$$

where

$$\mathbf{B}_i = \left[\beta_{ki}\right] = \frac{\vec{\phi}_i^T\mathbf{M}}{\vec{\phi}_i^T\mathbf{m}\,\mathbf{e}^\mathbf{s}}, \quad \Gamma_i = \frac{\vec{\phi}_i^T\mathbf{m}\mathbf{e}^\mathbf{s}}{\vec{\phi}_i^T\mathbf{m}\vec{\phi}_i}. \tag{15}$$

In case of deterministic excitation a novel formulation can be applied in order to use solution methods that are well known for the special case of uniform support excitation. By defining a non-dimensional ground acceleration vector,

$$\mathbf{F_g}^\mathbf{T}(t) = \begin{bmatrix} \ddot{u}_{g1}/\ddot{u}_{gref} & \ddot{u}_{g2}/\ddot{u}_{gref} & \cdots & 1 & \cdots & \ddot{u}_{gM}/\ddot{u}_{gref} \end{bmatrix}, \tag{16}$$

where $\ddot{u}_{gref} \neq 0$ represents a reference acceleration component, Eq. (12)is transformed to

$$\mathbf{m}\,\ddot{\mathbf{u}} + \mathbf{c}\,\dot{\mathbf{u}} + \mathbf{k}\,\mathbf{u} = -\mathbf{m}(-\mathbf{k}^{-1}\mathbf{k_g})\ddot{\mathbf{u}}_\mathbf{g} = -\mathbf{m}\,\mathrm{E}(t)\ddot{u}_{gref}, \tag{17}$$

with the time-dependent influence vector

$$\mathbf{E}(t) = (-\mathbf{k}^{-1}\mathbf{k_g})\mathbf{F_g}(t). \tag{18}$$

When comparing Eq. (17) to Eq. (2) of the beam under uniform support excitation it turns out that both are of the same dimension and structure. Thus, the modal equations, Eq. (14), become

$$\ddot{y}_i + 2\zeta_i\omega_i\dot{y}_i + \omega_i^2 y_i = -\Gamma_i(t)\ddot{u}_{gref} \tag{19}$$

with the time-dependent participation factor

$$\Gamma_i(t) = \frac{\vec{\phi}_i^T \mathbf{m}(-\mathbf{k}^{-1}\mathbf{k_g})}{\vec{\phi}_i^T \mathbf{m}\vec{\phi}_i} \mathbf{F_g}(t). \tag{20}$$

5 STATIONARY RANDOM EXCITATION

5.1 An approximate excitation model

Under close examination of the seismic analysis of multiply supported bridge structures subjected to spatially varying ground motion three main effects have to be taken into account, compare [9]:

(a) Wave passage, considering the difference in the arrival times of the waves at stations located apart due to the finite nature of the seismic wave velocities,
(b) Incoherence, caused due to wave propagation in a heterogeneous medium with numerous reflections and refractions,
(c) Site response, considering local soil conditions.

The present paper introduces an approximate procedure, the Pseudo Excitation Method (PEM),see e.g. [8], which includes both the cross-correlation terms between the participant modes and between the excitations.

Local effects are treated by assuming different power spectral densities (PSDs) of the ground acceleration at each support,

$$S_{\ddot{u}_{gk}\ddot{u}_{gk}}(\omega) = \lambda_k S_{\ddot{u}_{g1}\ddot{u}_{g1}}(\omega) = \lambda_k S_a(\omega), \tag{21}$$

where it is suggested that the factor λ_k can be estimated by the ratio of individual mean square values,

$$\lambda_k = \frac{\int_0^{\omega_1} S_{\ddot{u}_{gk}\ddot{u}_{gk}}(\omega)d\omega}{\int_0^{\omega_1} S_{\ddot{u}_{g1}\ddot{u}_{g1}}(\omega)d\omega} = \frac{\int_0^{\omega_1} S_{\ddot{u}_{gk}\ddot{u}_{gk}}(\omega)d\omega}{\int_0^{\omega_1} \lambda_1 S_a(\omega)d\omega}. \tag{22}$$

In the next step, this random excitation is replaced by a pseudo sinusoidal excitation, where the first ground node is taken as reference node,

$$\ddot{u}_{g1}(t) = \sqrt{\lambda_1 S_a(\omega)}\exp(i\omega t). \tag{23}$$

The time delay of the motion of ground node j, when measured relative to the reference node 1, reads

$$T_j = \left(u_{gj} - u_{g1}\right)/v_{app} \tag{24}$$

where v_{app} denotes the surface apparent wave velocity. Finally, the vector of pseudo sinusoidal excitation becomes

$$\ddot{\tilde{\mathbf{u}}}_{\mathbf{g}} = \ddot{\tilde{\mathbf{U}}}_{\mathbf{g}} \exp(i\omega t) = \mathbf{d}\sqrt{\lambda_j S_a(\omega)}\exp(i\omega t), \tag{25}$$

with the non-dimensional complex vector

$$\mathbf{d} = \begin{bmatrix} 1 & \exp(-i\omega T_2) & \cdots & \exp(-i\omega T_M) \end{bmatrix}^T. \tag{26}$$

5.2 Computation of structural response

The use of PEM makes it possible to determine the PSDs of the dynamic response. Thereby the vector of the total response is formulatedby means of a time-harmonic Ansatz,

$$\tilde{\mathbf{u}}^{\mathbf{t}}(t) = \tilde{\mathbf{u}}(t) + \tilde{\mathbf{u}}^{\mathbf{s}}(t) = \left[\tilde{\mathbf{U}}(i\omega) + \tilde{\mathbf{U}}^{\mathbf{s}}(i\omega)\right]\exp(i\omega t). \tag{27}$$

Solving the equation of motion associated to $\tilde{\mathbf{u}}(t)$, compare Eq. (12),

$$\mathbf{m}\ddot{\tilde{\mathbf{u}}} + \mathbf{c}\dot{\tilde{\mathbf{u}}} + \mathbf{k}\tilde{\mathbf{u}} = -\mathbf{M}\ddot{\tilde{\mathbf{u}}}_{\mathbf{g}} \tag{28}$$

gives the complex amplitude vector of the dynamic part of nodal displacements

$$\tilde{\mathbf{U}}(i\omega) = -\mathbf{H}(i\omega)\mathbf{M}\mathbf{d}\sqrt{\lambda_j S_a(\omega)}, \tag{29}$$

where the complex transfer matrix is defined as

$$\mathbf{H}(i\omega) = \left[\mathbf{k} + i\omega\mathbf{c} - \omega^2\mathbf{m}\right]^{-1}. \tag{30}$$

The pseudo-static contribution, see Eqs. (7) and Eq. (25), becomes

$$\tilde{\mathbf{U}}^{\mathbf{s}}(i\omega) = -\mathbf{k}^{-1}\mathbf{k}_{\mathbf{g}}\tilde{\mathbf{U}}_{\mathbf{g}} = \frac{1}{\omega^2}\mathbf{k}^{-1}\mathbf{k}_{\mathbf{g}}\ddot{\tilde{\mathbf{U}}}_{\mathbf{g}} = \frac{\sqrt{\lambda_j S_a(\omega)}}{\omega^2}\mathbf{k}^{-1}\mathbf{k}_{\mathbf{g}}\mathbf{d}. \tag{31}$$

Finally the total pseudo structural displacement vector reads

$$\tilde{\mathbf{u}}^{\mathbf{t}}(t) = \tilde{\mathbf{U}}^{\mathbf{t}}(i\omega)\exp(i\omega t), \qquad \tilde{\mathbf{U}}^{\mathbf{t}}(i\omega) = \left[\frac{1}{\omega^2}\mathbf{k}^{-1}\mathbf{k}_{\mathbf{g}} - \mathbf{H}(i\omega)\mathbf{M}\right]\mathbf{d}\sqrt{\lambda_j S_a(\omega)}, \tag{32}$$

and the corresponding matrix of PSDs can be expressed as

$$\left[\mathbf{S}_{\tilde{\mathbf{u}}^{\mathbf{t}}\tilde{\mathbf{u}}^{\mathbf{t}}}(\omega)\right] = \left[\tilde{\mathbf{U}}^{\mathbf{t}*}\right]\left[\tilde{\mathbf{U}}^{\mathbf{t}}\right]^T, \tag{33}$$

where the superscript * represents the complex conjugate of the vector.

6 CONCLUSIONS

A new formulation for linear elastic multi-span beams under multiple support excitation has been proposed in order to reduce the degrees of freedom in a mechanically consistent manner. The resulting differential equations are formally identical to those of structures under uniform support excitations. Thus, especially in case of deterministic excitation it becomes possible to apply only slightly modified procedures for treating vibrations of structures under uniform support excitation.Making use of modal analysis, e.g., it becomes necessary to introduce time-dependent participation factors.

For stationary random multiple support excitation an approximate procedure, the Pseudo Excitation Method is introduced, which includes the main effects of wave passage and site response.

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