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DICAM

SIMULATION OF HIGHER-ORDER STOCHASTIC PROCESSES BY SPECTRAL REPRESENTATION: ASYMMETRIC PROCESSES

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Abstract.*An extension of the classical spectral representation method for asymmetrically non-Gaussian stochastic processes is proposed. To achieve this, new orthogonal increments in the Cramér spectral representation are proposed which satisfy the necessary orthogonality conditions up to 3rd order. Two new quantities are defined to facilitate the expansion, the pure power spectrum and partial bicoherence, whichare used to decouple the wave components into their individual and coupled contributions to the process. The method is used tosimulate time histories of the wind pressure coefficients consistent with wind tunnel test data such thatthe estimated power spectrum and bispectrum of the generated samplesconverge to the empiricaltargetspectra respectively.*

1 INTRODUCTION

Stochastic processesare widely applied to model random physical phenomenafrom heterogeneous material structures to structural excitations including wind and earthquake time histories.Solving problems that involve these stochastic models requires generation of samples of stochastic processes. The accuracy of these realizations (i.e. their ability to match the real properties of the random phenomenon) is critical to the credibility of the simulation-based approaches. In general, simulation is made possible byexpandingthe process as a finite sum

$$x(t,\theta) = \sum_{i=1}^{N} C_i(\theta)\,\xi_i(t) \tag{1}$$

where $C_i(\theta)$ are random variables (usually treated as independent) and $\xi_i(t)$ are deterministic basis functions.

The Spectral Representation Method [1,2,3] and Karhunen-Loève expansion [4,5] are the most commonly employed expansionscorresponding to the cases where the basis functions are the harmonics and the eigenfunctions of the covariance kernel, respectively.Both methods typically treat the random variables $C_i(\theta)$ as independent with the result that the expanded process can possess the correct properties only up to 2ndorder, its covariance, and resulting in an asymptotically Gaussian process. To overcome thislimitation, nonlinear transformations are typically utilized to match the marginal non-Gaussian PDF exactlyand match the covariance function asclosely as possible [6]. Recently, methods for efficiently generating thesetranslation processes have been developed for stationary [7,8] and non-stationary [9,10] processes using the spectral representation method and the Karhunen-Loève expansion.Related methods have been developed as well using, for example, polynomial chaos expansion based nonlinear transformations [11] and schemes that iteratively update the marginal densities of the independent random variables in Karhunen-Loève expansion [12,13], but these methodologies cannot match the marginal non-Gaussian PDF exactly.

Though the aforementionedmethods generatesampleswith the correctnon-Gaussian marginal probability densities and covariance, theyremain inherently 2nd-order in their correlation structure. In this regard, by deriving a higher-order stochastic representation, the correlation structure of the process,up to 3rdorder, will be directly incorporated into the expansion. To achieve this, neworthogonal increments in the Cramér spectral representation [14] are proposed that satisfy orthogonal conditions up to 3rd-order.Along the way, we define two new quantities referred to as the pure power spectrum and partial bicoherence that facilitate this expansion. We then apply the proposed expansion for generation of skewed wind pressure coefficient time histories consistent with existing wind tunnel test data.

2 CUMULANTS AND POLYSPECTRA

Higher-order stochastic processes possess certain properties that can be described through theirmoments and cumulants of various order. The joint moment of order $r = k_1 + k_2 + \cdots + k_n$of a real random vector $X = \{x_1, x_2, \cdots x_n\}$ are defined as [15,16,17]

$$
\begin{aligned}
m_{k_1,k_{2,\cdots,}k_n} &= \frac{1}{i^r}\frac{\partial^r \Phi(\omega_1,\omega_2,\cdots,\omega_n)}{\partial\omega_1^{k_1}\partial\omega_2^{k_2}\cdots\partial\omega_n^{k_n}}\bigg|_{\omega_1+\omega_2+\cdots\omega_n=0} \\
c_{k_1,k_{2,\cdots,}k_n} &= \frac{1}{i^r}\frac{\partial^r \ln\Phi(\omega_1,\omega_2,\cdots,\omega_n)}{\partial\omega_1^{k_1}\partial\omega_2^{k_2}\cdots\partial\omega_n^{k_n}}\bigg|_{\omega_1+\omega_2+\cdots\omega_n=0}
\end{aligned}
\tag{2}
$$

where $\Phi(\cdot)$ is characteristic function of random vector X. Furthermore, let$f(t)$ be a real stationary stochastic process with zero mean, the cumulants of the process can be related to the moments as

$$
\begin{aligned}
c_1^f = m_1^f = 0, \quad & c_2^f(\tau) = m_2^f(\tau), \quad c_3^f(\tau_1,\tau_2) = m_3^f(\tau_1,\tau_2) \\
c_4^f(\tau_1,\tau_2,\tau_3) &= m_4^f(\tau_1,\tau_2,\tau_3) - m_2^f(\tau_1)m_2^f(\tau_2-\tau_3) \\
&\quad - m_2^f(\tau_2)m_2^f(\tau_3-\tau_1) - m_2^f(\tau_3)m_2^f(\tau_1-\tau_2) \\
&\vdots
\end{aligned}
\tag{3}
$$

For Gaussian processes, the cumulants of order higher than twoare identically zero. Thus non-zero cumulants provide one measure of deviation from Gaussian.

The nth order polyspectrum of a stationary stochastic process is defined as the Fourier transform of the nth order cumulant[15]

$$C_n^x(\omega_1,\cdots,\omega_{n-1}) = \frac{1}{(2\pi)^{n-1}} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} c_n^x(\tau_1,\cdots,\tau_{n-1}) e^{-i(\omega_1\tau_1+\cdots+\omega_{n-1}\tau_{n-1})} d\tau_1 \cdots d\tau_{n-1} \tag{4}$$

where $C_n^x(\omega_1,\cdots,\omega_{n-1})$ is, in general, complex. The classical power spectrum is the lowest order polyspectrum defined as the Fourier transform of 2nd order cumulant. The bispectrum can be expressed in terms of the 3rd order cumulantusing Eq. (4) as

$$C_3^x(\omega_i,\omega_j) = B(\omega_i,\omega_j) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c_3^x(\tau_i,\tau_j) e^{-i(\omega_i\tau_j+\omega_i\tau_j)} d\tau_i \, d\tau_j \tag{5}$$

and represents certain asymmetric properties of non-Gaussian processes. In particular, the real and imaginary parts of the bispectrum relate to the skewness of the stochastic process and its derivative, respectively[18,19]. The skewness ofthe process be obtained by integrating the real parts of bispectrum, and the skewness of derivative of the processobtained by integrating the imaginary components as

$$\begin{aligned} \mathbb{E}[f(t)^3] &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Re B(\omega_i,\omega_j) d\omega_i \, d\omega_j \\ \mathbb{E}\left[\left(\frac{\partial f(t)}{\partial t}\right)^3\right] &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Im B(\omega_i,\omega_j) d\omega_i \, d\omega_j \end{aligned} \tag{7}$$

Lastly, the bispectrum can be expressed in terms of its amplitude and biphase [20] as

$$B(\omega_i,\omega_j) = |B(\omega_i,\omega_j)| e^{i\beta(\omega_i,\omega_j)} \tag{8}$$

where

$$\beta(\omega_i,\omega_j) = \tan^{-1}\left[\frac{\Im B(\omega_i,\omega_j)}{\Re B(\omega_i,\omega_j)}\right] \tag{9}$$

Polyspectra are hierarchical in nature, meaning that each higher-order spectrum contributes to the lower-order spectra. To quantify the proportion of the power spectrum that derives from the bispectrum, we define a partial bicoherenceand a corresponding pure power spectrum as

$$b_p^2(\omega_i,\omega_j) = \frac{|B(\omega_i,\omega_j)|^2}{S_p(\omega_i) S_p(\omega_j) S(\omega_i+\omega_j)} \tag{10}$$

where

$$S_p(\omega_k) = S_p(\omega_k)\left[1 - \sum_{i+j=k}^{i\geq j\geq 0} b_p^2(\omega_i,\omega_j)\right] \tag{11}$$

The partial bicoherence isolates the fraction of power in the wave with frequency $\omega_k$that results from interactions of the waves with frequency ω_i and ω_j having $\omega_i+\omega_j=\omega_k$. The partial bicoherence has the following property

$$0 \leq \sum_{i+j=k}^{i\geq j\geq 0} b_p^2(\omega_i,\omega_j) \leq 1 \tag{12}$$

where the summation considers the interaction of all pairs of frequencies with $\omega_i + \omega_j = \omega_k$ such that $\sum_{i+j=k}^{i\geq j\geq 0} b_p^2(\omega_i, \omega_j) = 1$ means that all power at frequency ω_k arises from wave interactions and $\sum_{i+j=k}^{i\geq j\geq 0} b_p^2(\omega_i, \omega_j) = 0$ means that no power comes from interactions.

3 SPECTRAL REPRESENTATION FOR HIGHER-ORDER PROCESSES

The Cramér spectral representation [14] expands a general zeromean, real stationary processes $f(t)$ in the form of the following Fourier-Stieltjes integral

$$f(t) = \int_{-\infty}^{\infty} \cos(\omega t)\, du(\omega) \tag{13}$$

with the orthogonal increments, $du(\omega)$ and $dv(\omega)$,satisfying the followingorthogonal conditions [21, 22]

$$\begin{gathered} E[du(\omega)] = E[dv(\omega)] = 0 \\ E\big[du(\omega_i)du(\omega_j)\big] = E\big[dv(\omega_i)dv(\omega_j)\big] = \delta(\omega_i - \omega_j)2S(\omega_i, \omega_j)d\omega_i d\omega_j \\ E\big[du(\omega_i)du(\omega_j)du(\omega_k)\big] = \delta(\omega_i + \omega_j - \omega_k)2\Re B(\omega_i, \omega_j)d\omega_i d\omega_j \\ E\big[dv(\omega_i)dv(\omega_j)dv(\omega_k)\big] = -\delta(\omega_i + \omega_j - \omega_k)2\Im B(\omega_i, \omega_j)d\omega_i d\omega_j \\ \vdots \end{gathered} \tag{14}$$

where is the $\delta(\cdot)$ Dirac delta function.

The Spectral Representation Method, firstintroduced by Rice[23] and further developed by Shinozuka and Deodatis [1-3,24],exploits the Cramér spectral representation to simulate 2nd order stationary processes by assigning the following orthogonal increments:

$$\begin{aligned} du(\omega_k) &= \sqrt{2}\sqrt{2S(\omega_k)\Delta\omega_k}\cos(\varphi_k) \\ dv(\omega_k) &= \sqrt{2}\sqrt{2S(\omega_k)\Delta\omega_k}\sin(\varphi_k) \end{aligned} \tag{15}$$

where the random phase anglesφ_k are independentand uniformly distributed in the range $[0, 2\pi]$. The prescribed orthogonal increments yieldthe followingexpansionfor stationary and Gaussian stochastic processes

$$f(t) = \sqrt{2}\sum_{k=1}^{\infty} \sqrt{2S(\omega_k)\Delta\omega_k} \cos(\omega_k t \tag{16}$$

Weaim to develop a higher-order spectral representation method by deriving new, higher-order orthogonal increments.This starts by defining adiscretized form of partial bicoherence in Eq. (10) as

$$b_p^2(\omega_i, \omega_j) = \frac{|B(\omega_i, \omega_j)|^2 \Delta\omega_i^2 \Delta\omega_j^2}{S_p(\omega_i)\Delta\omega_i S_p(\omega_j)\Delta\omega_j S(\omega_i + \omega_j)\Delta(\omega_i + \omega_j)} \tag{17}$$

Next, consider that the orthogonal increments can be decomposed according to their pure and interaction components as

$$\begin{aligned} du(\omega_k) &= du_p(\omega_k) + du_I(\omega_k) \\ dv(\omega_k) &= dv_p(\omega_k) + dv_I(\omega_k) \end{aligned} \tag{18}$$

with each term defined as

$$du_p(\omega_k) = \sqrt{2}\sqrt{2S_P(\omega_k)\Delta\omega_k}\cos(\varphi_k)$$
$$dv_p(\omega_k) = \sqrt{2}\sqrt{2S_P(\omega_k)\Delta\omega_k}\sin(\varphi_k) \tag{19}$$

and

$$du_I(\omega_k) = \sqrt{2}\sqrt{2S_P(\omega_k)\Delta\omega_k}\left\{\sum_{i+j=k}^{i\geq j\geq 0}\left|b_p(\omega_i,\omega_j)\right|\cos\left[\varphi_i+\varphi_j+\beta(\omega_i,\omega_j)\right]\right\}$$
$$dv_I(\omega_k) = \sqrt{2}\sqrt{2S_P(\omega_k)\Delta\omega_k}\left\{\sum_{i+j=k}^{i\geq j\geq 0}\left|b_p(\omega_i,\omega_j)\right|\sin\left[\varphi_i+\varphi_j+\beta(\omega_i,\omega_j)\right]\right\} \tag{20}$$

and $\beta(\omega_i,\omega_j)$ is the biphase from Eq. (9). The pure components, Eq. (19), correspond to the orthogonal increments for the original spectral representation method, exceptbased on pure power spectrum in Eq. (11). Meanwhile, interaction increments, Eq. (20), are newly generated from the bispectrum of the process.Integrating these higher-order increments into the Cramér spectral representation yields the following expansion

$$\begin{aligned} f(t) = {} & \sqrt{2}\sum_{k=1}^{\infty}\sqrt{2S(\omega_k)\Delta\omega_k}\cos(\omega_k t-\varphi_k) \\ & +\sqrt{2}\sum_{k=1}^{\infty}\sum_{i+j=m}^{i\geq j\geq 0}\sqrt{2S(\omega_k)\Delta\omega_k}\left|b_p(\omega_i,\omega_j)\right|\cos\left[(\omega_i+\omega_j)t-(\varphi_i+\varphi_j+\beta(\omega_i,\omega_j))\right] \end{aligned} \tag{21}$$

that produces asymmetric third-order stochastic processes. The spectral representation in Eq. (21), named as bispectral representation method (BSRM), possesses both the prescribed power spectrum and bispectrum. Furthermore, the proposed orthogonal increments satisfy the orthogonal properties of Cramér spectral representation up to 3rd order as shown in [25].

4 WIND PRESSURE SIMULATION

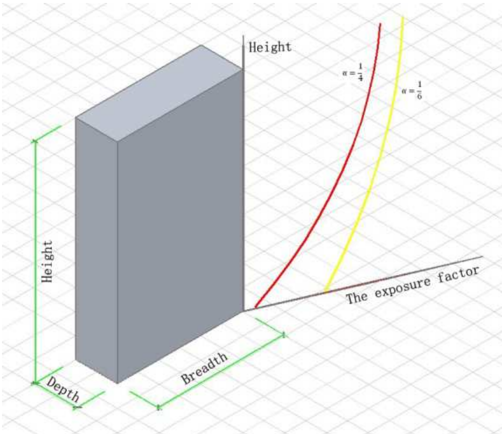
A series of wind tunnel tests have been conducted by Tokyo Polytechnic University as one part of the Wind Effects on Buildings and Urban Environment, the 21st Century Center of Excellence Program (2003-2007) with data available online [26]. One component of this testing program considers wind pressures on high-rise buildings. In the database, twenty two1/400 scale models of high-rise building were considered, and the time series data of point wind pressure coefficients were estimated for 394 test cases with different geometric building parameters,exposure factors, and wind direction angles.

We consider one such structure with parameters shown in Table 1. In this test, data are collected at 443 measurement locations on each surfaces of the model in the windward, right-sideward, leeward, and left-sideward directions. The statistical distributions of wind pressure are shown in Figure 2 for this structure illustrating that the wind pressures are clearly non-Gaussian at many locations. In these analyses, a single time histories of duration $T = 32.767$ sec. with $\Delta t = 0.001$ sec. is divided into 8 individual samples of duration $T = 4$ sec. and the power spectra and bispectra estimated.

Breadth : Depth : Height	0.1m : 0.1m : 0.5m
Exposure factor	1/4
Wind direction angle	0°

Table 1 : Wind-tunnel test parameters for the considered structure.

a)



b)

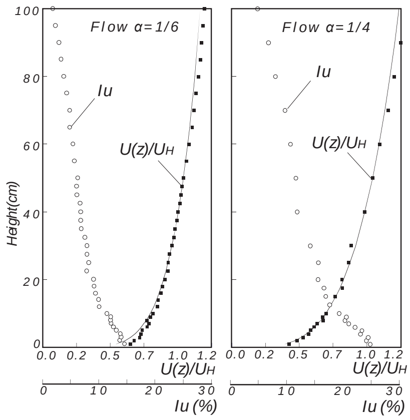
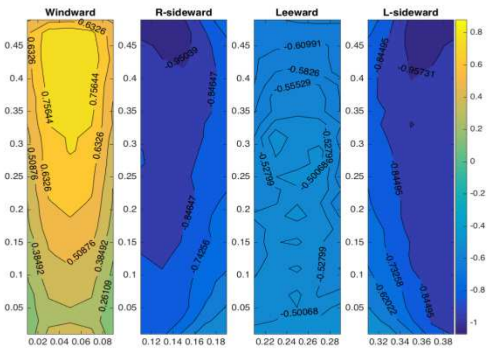
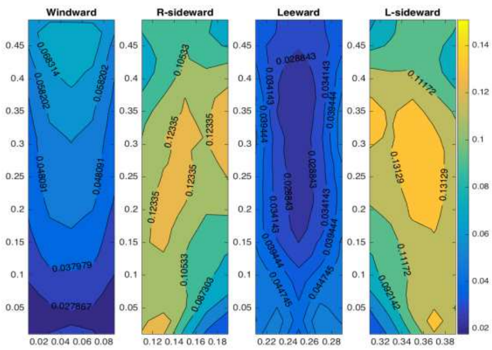


Figure 1:High-rise building model and incoming flow in wind tunnel test a) definitions of geometric parameters of building. b) vertical profile of incoming flow[26].

a)



b)



c)

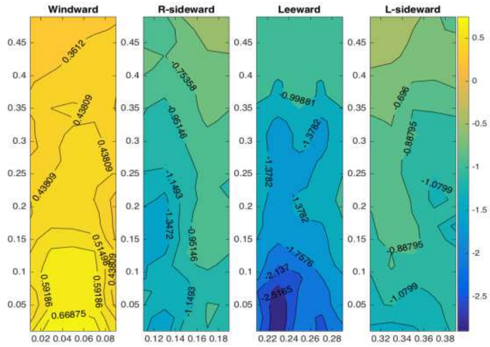
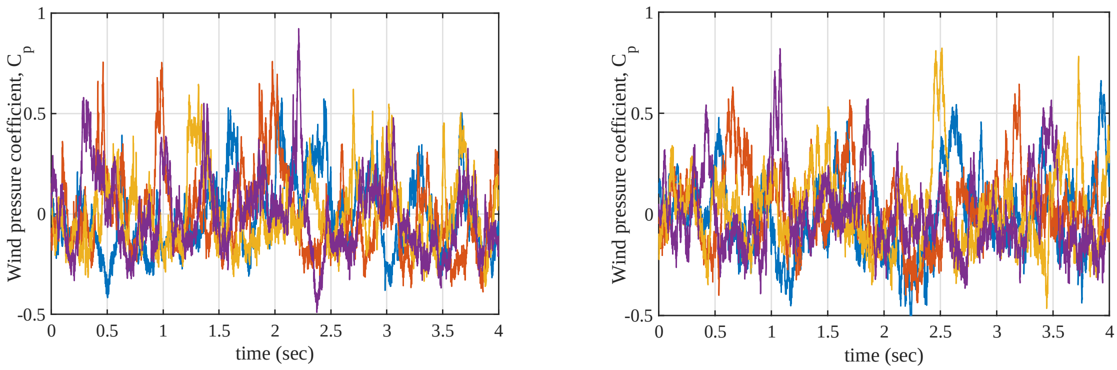


Figure 2: Contours of wind pressure coefficient statistics a) mean b) variance c) skewness.

The time history of the measurement point at the center-bottom of the windward surface has the highest skewness (0.7133).For this reason, the time history at this point is examinedhere. Using the estimated pure power spectrum and discretized partial bicoherence in Eq. (11) and (17), the proposed expansion is used to generated10,000 sample time histories of the wind pressure coefficient at this location. The target time histories and several time histories generated using Eq. (21) with duration $T = 4$sec. are shown in Figure 3.



a) b)

Figure 3: Time histories of wind pressure coefficient a) wind data b) BSRM.

From Figure 3, the time histories generated using the proposed method are qualitatively similar to those from the test with apositive skewness.The empirical PDF, power spectrum, and bispectrum of the generated time histories are compared with those estimated from the wind tunnel data ("target") in Figure 4 and the corresponding statistics are compared in Table 2.These results point to the method's ability to accurately match 3^{rd}-order properties of the process. Not only is the power spectrum of target and BSRM histories identical (as in a 2^{nd} order expansion), the real and imaginary bispectrum of BSRM samples also match the target ones from the wind pressure data.

	Target	BSRM
Variance	0.0324	0.0325
Skewness	0.6833	0.6301
Kurtosis	3.3264	3.0125

Table 2 : Statistics of target and BSRM-generated wind pressure coefficient time histories.

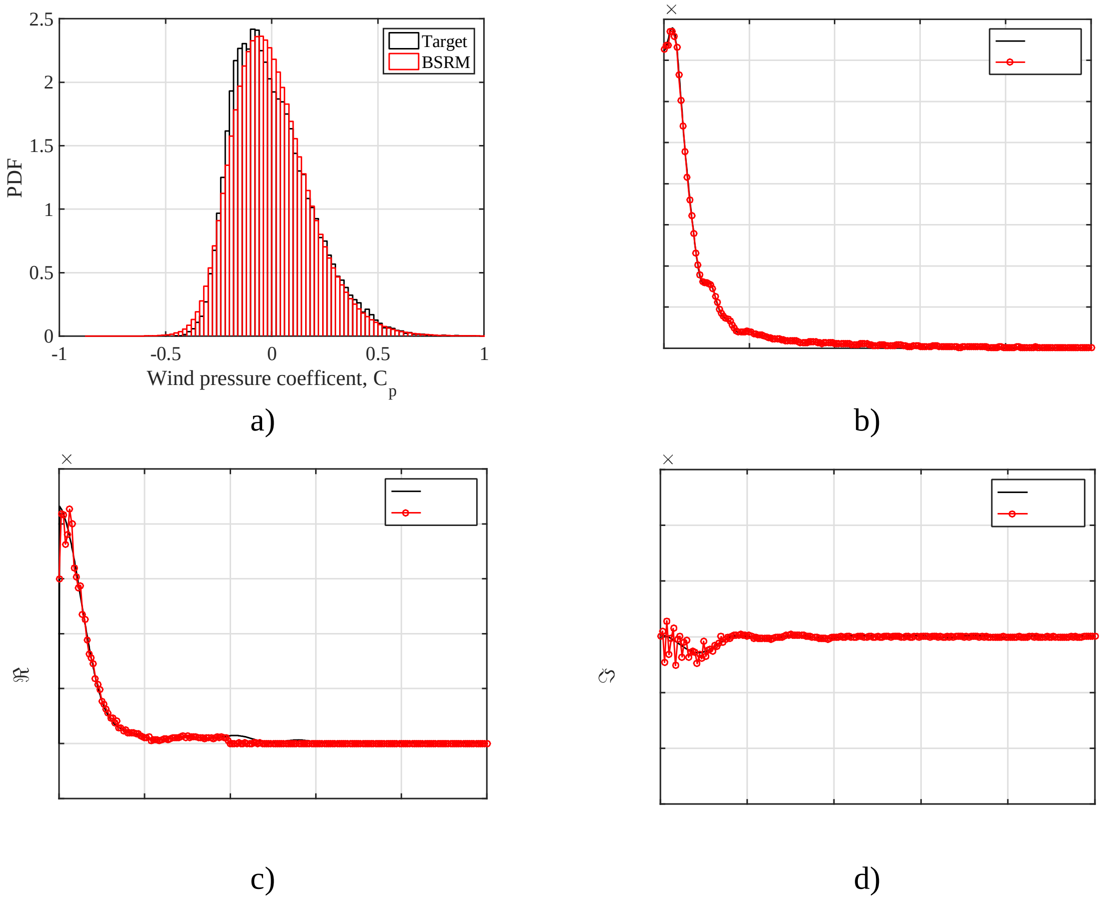


Figure 4: Comparisons between target and BSRM time histories a) empirical PDF b) power spectrum c) real bispectrum at 1 Hz. d) imaginary bispectrum at 1Hz.

12 CONCLUSION

In this paper, an advanced stochastic simulation method for generating asymmetric non-Gaussian processes has beenderived from the Cramér spectral representation. This represents a third-order extension of the original spectral representation method. By defining new third-order orthogonal increments, the methodology includes contributions from the bispectrum.To achieve this, two new quantities – the pure power spectrum and partial bicoherence – are defined to separate the independent and interacting components in power spectrum. Using this method, wind pressure time histories are generated to be consistent with observations from wind tunnel tests.

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