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DICAM

NONLINEAR RANDOM VIBRATIONS OF PLATES ENDOWED WITH FRACTIONAL DERIVATIVE ELEMENTS

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Abstract. *This paper deals with the problem of determining the nonlinear response of a plate endowed with fractional derivative elements and exposed to random loads. It is shown that an approximate solution of the nonlinear fractional partial differential equation governing the plate vibrations can be obtained via a statistical linearization based approach. The approach is implemented by employing a time-dependent representation of the response involving the eigen-functions of the linear problem. This representation allows deriving a nonlinear fractional differential equation governing the variation of the time-dependent part of the response, which is linearized in a mean square sense. A Boundary Element Method is implemented for conducting relevant Monte Carlo simulations. The simulation is done in conjunction with a Newmark integration scheme for calculating the response in the time domain induced by spectrum compatible realizations of the excitation. Comparisons between the approaches establish the reliability of the proposed linearization scheme.*

Sommario. *La memoria affronta il problema del calcolo della risposta di una piastra nonlineare sollecitata da un carico aleatorio e comprendente un elemento frazionario. Si dimostrate che le equazioni che governano le oscillazioni della piastra possono essere risolte approssimativamente mediante una tecnica di linearizzazione statistica. Tale metodologia è implementata utilizzando una rappresentazione della risposta dipendente dal tempo mediante le auto-funzioni del corrispondente problema lineare. Si deriva una equazione differenziale frazionaria nonlineare che governa la parte della risposta dipendente dal tempo e risolta tramite linearizzazione statistica. La memoria sviluppa anche una metodologia basata sul metodo degli elementi al contorno e sul metodo di Newmark per condurre simulazioni di tipo Monte Carlo. Il confronto numerico conferma l'affidabilità dell'approccio proposto.*

1 INTRODUCTION

Currently, fractional calculus is employed in a number of disciplines as diverse as electrical engineering, chemistry and biology [1]. Structural mechanics has also taken advantage of fractional calculus. Indeed, it has become a quite established tool for describing the viscoelastic behaviour of materials since the pioneering works of Nutting [2] and Gemant [3], and the theoretical contribution of Bagley and Torvik [4]. The review article by Rossikhin and Shitikova [5] provides a broad view on the use of fractional calculus in solid mechanics considering a number of problems involving single-degree-of-freedom oscillators, multi-degree of freedom systems, beams and plates excited by deterministic loads.
In this paper, the problem of determining the large displacements of a nonlinear plate endowed with a fractional derivative element and excited by a random load is addressed. An approximate statistical linearization solution is developed for estimating the response statistics and its reliability is assessed against relevant Monte Carlo data.

2 PRELIMINARY REMARKS ON FRACTIONAL OPERATORS

A number of representations, all generalizing the operators of differentiation and of integration, are available in the open literature [1]. In this paper, consider the representations of Riemann-Liouville (RL) and Grünwald-Letnikov (GL) which are widely used in the context of visco-elasticity.
The important concept underlining the definition of fractional derivative relates to the definition of fractional integral which is obtained as a convolution of a function $w(t)$ with a power law kernel. That is,

$$ {}_0D_t^{-\gamma} = \frac{1}{\Gamma(\gamma)} \int_0^t \frac{w(\tau)}{(t-\tau)^{-\gamma+1}} d\tau \text{, for } \gamma>0, \tag{1} $$

with $\Gamma(\gamma)$ being the Gamma function. Clearly, for integer values of the power law $\gamma = n$ the Gamma function renders the factorial of the integer number, and thus eq. (1) provides the classical n-fold integral. The RL fractional derivative is constructed by differentiating eq. (1) m times. That is,

$$ {}_0^{RL}D_t^{\gamma} = \frac{1}{\Gamma(m-\gamma)} \frac{d^m}{dt^m} \int_0^t \frac{w(\tau)}{(t-\tau)^{\gamma+1-m}} d\tau \text{, } m\text{-}1\leq\gamma<m. \tag{2} $$

Therefore, the fractional derivative is calculated by first integrating the function (m-γ) times, and then by differentiating the result m times.
The GL representation [6] is given by the equation

$$ {}_0^{GL}D_t^{\gamma} w(t) = \sum_{k=0}^{m-1} \frac{w^{(k)}(0) t^{k-\gamma}}{\Gamma(k+1-\gamma)} + \frac{1}{\Gamma(m-\gamma)} \int_0^t \frac{w^{(m)}(\tau)}{(t-\tau)^{\gamma+1-m}} d\tau \text{, } m\text{-}1<\gamma<m. \tag{3} $$

Such a representation affords the implementation of algorithms for the numerical computation of fractional derivatives. Indeed, the series in eq. (3) can be expanded and the following series representation of the GL derivative can be derived:

$$ {}_0^{GL}D_t^{\gamma} w(t) = \lim_{\Delta t \to 0} \Delta t^{-\gamma} \sum_{k=0}^{n} GL_k w(t-k\Delta t) , \tag{4} $$

where GL_k are calculated recursively by the relationship

$$ GL_k = \frac{k-\gamma-1}{k} GL_{k-1} ;\ GL_0 = 1 . \tag{5} $$

Eq. (4) describes the G1-algorithm that is used in this paper for treating numerically terms with fractional derivatives. Such an algorithm reflects readily the fading memory property of the fractional derivative through the quantity $(k\text{-}\gamma\text{-}1)/k < 1$.

3. LARGE PLATE DISPLACEMENTS

3.1 Equations governing the large plate displacements

Consider a rectangular plate of sides a and b, with mass density ρ, thickness h, Young modulus E, and flexural stiffness D. The plate is exposed to a transverse load $q=q(x,y,t)$ which depends on the space coordinates (x,y), and on the time variable t and is endowed with a fractional derivative element of order γ and constant damping μ. Then, its transverse displacement $u=u(x,y,t)$ is governed by the equation of motion

$$\rho h \frac{\partial^2 u}{\partial t^2} + \mu_0 \partial_t^\gamma u + D\nabla^4 u - h\left(\frac{\partial^2 \phi}{\partial y^2}\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 \phi}{\partial x^2}\frac{\partial^2 u}{\partial y^2} - 2\frac{\partial^2 \phi}{\partial x \partial y}\frac{\partial^2 u}{\partial x \partial y}\right) = q\,, \tag{6}$$

where $\nabla^4 = (\partial^2/\partial x^2 + \partial^2/\partial y^2 + 2\partial^2/\partial x \partial y)$ is the biharmonic operator, and $\phi = \phi(x,y,t)$ is the Airy stress function that is governed by the equation

$$\nabla^4 \phi = E\left[\left(\frac{\partial^2 u}{\partial x \partial y}\right)^2 - \frac{\partial^2 u}{\partial x^2}\frac{\partial^2 u}{\partial y^2}\right]. \tag{7}$$

The load is assumed of a separable kind, so that

$$q(x,y,t) = p(x,y)\,f(t)\,, \tag{8}$$

where $p(x,y)$ is a deterministic function, and $f(t)$ is a random process of a given power spectral density function $S(\omega)$ and with autocorrelation function

$$< f(t-\tau_1)\,f(t-\tau_2) >= \int_{-\infty}^{\infty} S(\omega)\exp[\imath\omega(\tau_2 - \tau_1)]d\omega. \tag{9}$$

Eq. (6) allows investigating the vibrations of an elastic plate into a viscous medium or on a viscoelastic foundation [5]. In this context, the fractional derivative operator allows introducing simultaneously stiffness and damping elements so that $\gamma = 0$ and $\gamma = 1$ represent the case, respectively, of a linear spring and of a viscous damper acting on the plate.

3.2 Approximate plate response determined by a statistical linearization-based approach

Currently, an exact solution of eq. (6) is unavailable. Therefore, an approximate solution is sought via a statistical linearization based approach [7]. For this purpose, the response of the system is represented by Galerkin expansions of the vertical displacement and of the stress function having time-dependent amplitudes. Specifically,

$$u = \sum_{m,n} w_{mn}(t) U_{mn}(x,y)\,, \tag{10}$$

and

$$\phi = \frac{P_x y^2}{2bh} + \frac{P_y x^2}{2ah} + \sum_{m,n} w_{mn}^{(2)}(t)\varphi_{mn}(x,y)\,, \tag{11}$$

with P_x and P_y being total tension loads applied, respectively, on the sides $x = 0$, a and $y = 0$, b of the plate and $\sum_{m,n} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}$. Further, the eigen-functions U_{mn} and φ_{mn} relate to the linear problem only, depend upon the specific boundary conditions and are orthogonal to each other.

That is,

$$\iint_A U_{mn} U_{kl} dA = \iint_A \varphi_{mn} \varphi_{kl} dA = \frac{4}{ab} \delta_{mk} \delta_{nl}, \tag{12}$$

with δ_{mn} denoting the Kronecker delta (1 for $m = n$, 0 otherwise), and A denoting the plate surface. Next introduce the quantities

$$R_{xx}(M,N,m,n) = \iint_A \frac{\partial^2 U_{mn}}{\partial x^2} U_{MN} dA \text{ and } R_{yy}(M,N,m,n) = \iint_A \frac{\partial^2 U_{mn}}{\partial y^2} U_{MN} dA, \tag{13}$$

Further, substituting eq. (10) and (11) into eq. (6) and (7), doing algebraic manipulations reflecting error projection in the space of eigen-functions, and observing that the stress function amplitudes $w^{(2)}_{mn}$ can be expressed in terms of w_{mn}, a nonlinear fractional ordinary differential equation for the time-amplitudes w_{mn} is found. Specifically,

$$\begin{aligned} &\ddot{w}_{MN} + \frac{\mu}{\rho h} {}_0D_t^\gamma w_{MN} + \omega_{MN}^2 w_{MN} - \frac{4}{ab\rho h}\left(\frac{P_x}{b} \sum_{m,n} w_{mn} R_{xx}(M,N,m,n) + \frac{P_y}{a} \sum_{m,n} w_{mn} R_{yy}(M,N,m,n) \right) \\ &- \frac{4}{ab} \frac{E}{\rho} \sum_{m,n} \sum_{k,l} \sum_{p,q} w_{mn} w_{kl} w_{pq} I(M,N,m,n,k,l,p,q) = \frac{4}{ab\rho h} P_{MN} f(t), \quad for\, M,N = 1,2,... \end{aligned} \tag{14}$$

where

$$\begin{aligned} I(M,N,m,n,k,l,p,q) = &\sum_{i,j} \iint_A \frac{\partial^2 U_{mn}}{\partial x^2} \frac{\partial^2 \varphi_{ij}}{\partial y^2} U_{MN} + \frac{\partial^2 U_{mn}}{\partial y^2} \frac{\partial^2 \varphi_{ij}}{\partial x^2} U_{MN} - 2 \frac{\partial^2 U_{mn}}{\partial x \partial y} \frac{\partial^2 \varphi_{ij}}{\partial x \partial y} U_{MN} dA \\ &\times \iint_A \frac{\partial^2 U_{kl}}{\partial x \partial y} \frac{\partial^2 U_{pq}}{\partial x \partial y} \varphi_{ij} - \frac{\partial^2 U_{kl}}{\partial x^2} \frac{\partial^2 U_{pq}}{\partial y^2} \varphi_{ij} dA \left[\iint_A \frac{\partial^4 \varphi_{ij}}{\partial x^2 \partial y^2} \varphi_{ij} + \frac{\partial^2 \varphi_{ij}}{\partial x^2 \partial y^2} \varphi_{ij} + 2 \frac{\partial^2 \varphi_{ij}}{\partial x \partial y} \varphi_{ij} dA \right]^{-1}, \end{aligned} \tag{15}$$

$$P_{MN} = \iint_A p(x,y) U_{MN} dA, \tag{16}$$

and ω_{MN} denotes the natural frequency of the linear plate. An approximate solution of eq. (14) is sought by replacing this nonlinear system by the equivalent linear system

$$\ddot{w}_{MN} + \mu/(\rho h) {}_0D_t^\gamma w_{MN} + \omega_{eq,MN}^2 w_{MN} = 4P_{MN} f(t)/(\rho hab), \text{ for } M, N = 1,2,\ldots \tag{17}$$

This linear system comprises an equivalent stiffness determined by minimizing the error between the nonlinear system and the linear one in a mean square sense. That is, it is found as the solution of the minimization problem

$$\partial < \varepsilon^2 > / \partial \omega_{eq,MN}^2 = 0, \text{ for } M, N = 1, 2,\ldots \tag{18}$$

where ε is the error between the linear and the nonlinear system given by the equation

$$\begin{aligned} \varepsilon = &\omega_{MN}^2 w_{MN} - \frac{4}{ab\rho h}\left(\frac{P_x}{b} \sum_{m,n} w_{mn} < \frac{\partial^2 U_{mn}}{\partial x^2}, U_{MN} > + \frac{P_y}{a} \sum_{m,n} w_{mn} < \frac{\partial^2 U_{mn}}{\partial y^2}, U_{MN} > \right) \\ &- \frac{4}{ab} \frac{E}{\rho} \sum_{m,n} \sum_{k,l} \sum_{p,q} w_{mn} w_{kl} w_{pq} I(M,N,m,n,k,l,p,q) - \omega_{eq,MN}^2 w_{MN}; \quad for\, M,N = 1,2,... \end{aligned} \tag{19}$$

Eq. (18) can be recast in the form

$$\begin{aligned} \omega_{eq,MN}^2 = &\omega_{MN}^2 - \frac{4}{ab\rho h} \frac{1}{P_{MN} S_{MN,MN}} \sum_{m,n} P_{mn} S_{MN,mn} \left[\frac{P_x}{b} R_{xx}(M,N,m,n) + \frac{P_x}{a} R_{yy}(M,N,m,n) \right] \\ &- \left(\frac{4}{ab} \right)^3 \frac{E}{\rho^3 h^2} \frac{1}{P_{MN} S_{MN,MN}} \sum_{m,n} \sum_{k,l} \sum_{p,q} P_{mn} P_{kl} P_{pq} (S_{MN,mn} S_{kl,pq} + S_{MN,kl} S_{mn,pq} + S_{MN,pq} S_{mn,kl}) \\ &\times I(M,N,m,n,k,l,p,q); \qquad for\, M,N = 1,2,... \end{aligned} \tag{20}$$

where

$$S_{MN,mn} = \int_{-\infty}^{\infty} H_{MN}(-\omega) S(\omega) H_{mn}(\omega) d\omega, \quad (21)$$

and $H_{MN}(\omega)$ is the transfer function associated with eq. (17). That is,

$$H_{MN}(\omega) = \left[-\omega^2 + \mu (i\omega)^\gamma / (\rho h) + \omega_{eq,MN}^2 \right]^{-1}. \quad (22)$$

After determining the equivalent stiffness, the response variance is readily determined via the equivalent linear system. Specifically, the variance is computed by the equation

$$\sigma^2(x,y) = < u^2(x,y) > = \left(\frac{4}{ab\rho h} \right)^2 \sum_{m,n} \sum_{k,l} P_{mn} P_{kl} U_{mn} U_{kl} S_{mn,kl} , \quad (23)$$

and the frequency spectrum of the transverse displacement at a certain point is computed by the equation

$$S_u(x,y,\omega) = \left(\frac{4}{ab\rho h} \right)^2 S(\omega) \sum_{m,n} \sum_{k,l} P_{mn} P_{kl} U_{mn} U_{kl} H_{mn}(-\omega) H_{kl}(\omega) . \quad (24)$$

3.3 Plate response estimated by Monte Carlo simulation

The numerical solution of the nonlinear fractional partial differential equations (6) - (7) is determined by a Boundary Element Method (BEM) based algorithm in the formulation proposed by Katsikadelis and Nerantzaki [8]. The key concept is to estimate the plate response via a time-varying representation of the solution of a classical linear BEM problem. Specifically, the approach is developed by considering the solution of the linear problems

$$\nabla^4 u = b_1(x,y,t), \quad (25)$$

and

$$\nabla^4 \varphi = b_2(x,y,t). \quad (26)$$

In this context, $b_1(x,y,t)$ and $b_2(x,y,t)$ are space-time dependent fictitious loads, which are identified by BEM.

The solution of the problem (25) has the integral representation [8]

$$\varepsilon u(P) = \int_A \Lambda_4 b_1 dA - \int_\Gamma \Lambda_1 u + \Lambda_2 u_n + \Lambda_3 \nabla^2 u + \Lambda_4 (\nabla^2 u)_n ds , \quad (27)$$

where $\varepsilon = 2\pi$ or π if the point P is inside the domain A or on the boundary Γ respectively, and the other quantities are given in Ref. [8].

Further, the equation

$$\varepsilon \nabla^2 u(P) = \int_A \Lambda_2 b_1 dA - \int_\Gamma \Lambda_1 \nabla^2 u + \Lambda_2 (\nabla^2 u)_n ds \quad (28)$$

holds. Eq. (27) and (28) can be used for estimating the unknown boundary quantities by introducing the associated boundary conditions. For this purpose, the plate domain and boundary are discretized and eq. (27) and (28) are collocated at boundary points. By doing so, the linear system of equations

$$\begin{bmatrix} [A_{11}] & [A_{12}] & [0] & [A_{14}] \\ [A_{21}] & [A_{22}] & [A_{23}] & [0] \\ [A_{31}] & [A_{32}] & [A_{33}] & [A_{34}] \\ [0] & [0] & [A_{43}] & [A_{44}] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{u_n\} \\ \{\nabla^2 u\} \\ \{(\nabla^2 u)_n\} \end{Bmatrix} = \begin{Bmatrix} \{B_1\} \\ \{B_2\} \\ \{0\} \\ \{0\} \end{Bmatrix} + \begin{bmatrix} 0 \\ 0 \\ [C_3] \\ [C_4] \end{bmatrix} [b_1] \quad (29)$$

is derived. The sub-matrices composing the first two rows are determined by the boundary conditions. The ones composing the last two rows are estimated from the discretized counterparts of eq. (27) and (28) via Gaussian integration over the domain and the boundary. System (29) allows determining the boundary quantities in terms of the fictitious load b_1. Thus, the response of the plate can be calculated by the equation:

$$\underline{u} = \underline{\underline{G}}_1 \underline{b}_1 , \tag{30}$$

where $\underline{\underline{G}}$ is a known matrix, $\underline{b}_1$ is a vector containing the values of the fictitious load at each point of the domain and $\underline{u}$ is a vector containing the response at that points. A similar procedure can be used for representing the stress function:

$$\underline{\phi} = \underline{\underline{G}}_2 \underline{b}_2 , \tag{31}$$

where it is observed that the only difference with the determination of $\underline{u}$ relates to the different boundary conditions.

The representation obtained in this manner is used for collocating the displacements and stress values into the original equations (6) - (7) to derive a set of fractional nonlinear ordinary differential equations for the fictitious loads $\underline{b}_1$ and $\underline{b}_2$. That is,

$$\rho h \underline{\underline{G}}_1 \ddot{\underline{b}}_1 + \mu \underline{\underline{G}}_1 {}_0\mathrm{D}_t^{\gamma} \underline{b}_1 + D \underline{b}_1 - F_1(\underline{\underline{G}}_{1,xx}, \underline{\underline{G}}_{2,xx}, \underline{\underline{G}}_{1,yy}, \underline{\underline{G}}_{2,yy}, \underline{\underline{G}}_{1,xy}, \underline{\underline{G}}_{2,xy}) = \underline{q} , \tag{32}$$

and

$$\underline{b}_2 = E F_2(\underline{\underline{G}}_{1,xx}, \underline{\underline{G}}_{1,yy}, \underline{\underline{G}}_{1,xy}) , \tag{33}$$

where $\underline{F}_1$ and $\underline{F}_2$ are nonlinear functions encapsulating the nonlinear elements of the original system.

The numerical solution of this fractional differential equation is obtained by a Newmark based algorithm implemented in conjunction with the G1 - algorithm of the GL fractional derivative. Specifically, the incremental equation of motion associated with eq. (32) is

$$\rho h \underline{\underline{G_1}} \cdot \Delta \ddot{\underline{b}}_1(t_i) + \mu \Delta t^{-\gamma} \underline{\underline{G_1}} \cdot \underline{P} + D \Delta \underline{b}_1(t_i) - \Delta \underline{F}_1(t_i) = \Delta \underline{q}(t_i) , \tag{34}$$

where

$$\underline{P} = \sum_{k=0}^{i-1} GL_k \Delta \underline{b}_1(t_i - k\Delta t) + GL_i \underline{b}_1(0) , \tag{35}$$

and GL_k are coefficients arising from the application of the G1 algorithm for the calculation of the fractional derivative [9]. Considering the fact that the fractional derivative calculation involves both present and past values of the response, this equation is recast as follows:

$$\begin{aligned} &\rho h \underline{\underline{G_1}} \cdot \Delta \ddot{\underline{b}}_1(t_i) + \mu \Delta t^{-\alpha} \underline{\underline{G}}_1 \cdot \Delta \underline{b}_1(t_i) + D \Delta \underline{b}_1(t_i) - \Delta \underline{F}_1(t_i) = \\ &= \Delta \underline{q}(t_i) - \mu \Delta t^{-\alpha} \underline{\underline{G}}_1 \cdot \left(\sum_{k=1}^{i-1} GL_k \Delta \underline{b}_1(t_i - k\Delta t) + GL_i \underline{b}_1(0) \right). \end{aligned} \tag{36}$$

This form of the incremental equation of motion can be used with a classical Newmark algorithm for calculating the response of a system. The calculations are done by incorporating only a limited number of terms in the computation of the past values. In this regard, numerical studies have shown that including about 200 past values is sufficient for obtaining a reliable estimate.

4 NUMERICAL RESULTS

Computations are discussed herein considering large vibrations of a square plate exposed to a uniform random load. In this context, the time-dependent part of the load is compatible with a coloured white noise spectrum given by the equation

$$\hat{S}(\varpi) = \frac{C\varpi^4}{[(\varpi^2 - k_1)^2 + (c_1\varpi)^2][(\varpi^2 - k_2)^2 + (c_2\varpi)^2]}, \tag{37}$$

with $\varpi = \omega/\omega_p$ being a normalized frequency spectrum, (ω_p denoting the peak frequency of the spectrum), and C, k_1, c_1, k_2 and c_2 being shape parameters. This particular spectrum can be regarded as the output of a cascade of two linear filters.

The geometric and material properties of the plate are summarized in Table 1, while the spectral parameters are shows in Table 2. In this regard, note that the excitation has a peak spectral period of 5 s and a standard deviation of 50 kN/m, while the quantity $p(x,y) = 1$. The numerical computations pertain to the case of simply supported stress – free edges investigated also by Katsikadelis and Nerantzaki [8].

$a = b$	h	E	ρ	ν	μ
10 m	0.1 m	2.1×10^{11} Pa	2355 kg/m^3	0.3	5×10^5 N/(m/s)$^\gamma$

Table 1. Geometrical and material properties of the plate considered in the numerical computations.

k_1	c_1	k_2	c_2
0.97	0.20	3.44	2.32

Table 2. Parameters of the normalized coloured white noise spectrum given by eq. (37).

Monte Carlo simulations and statistical linearization solutions are produced for the proposed case study in conjunction with various values of the fractional derivative order. The Monte Carlo data are obtained by synthesizing spectrum compatible realizations of the plate load and then implementing the BEM method with the Newmark algorithm. The numerical data are used for estimating the standard deviation of the plate response along the mid-span of the plate ($x = a/2$, $0 \le y \le b$) and the power spectral density function of response at the centre of the plate ($x = a/2$; $y = b/2$). The statistical linearization solution is estimated considering the appropriate eigen-functions associated with the boundary conditions.

Figures 1 and 2 show relevant numerical results. It is seen that the approximate solution is in good agreement with the numerical computation. The agreement is irrespective of the fractional derivative order. Further, not only the statistics but also the spectral content of the response is captured quite well over the entire frequency domain.

5 CONCLUDING REMARKS

This paper has developed an approximate approach for determining the nonlinear response of a plate endowed with a fractional derivative element and excited by a random load. The method is based on a statistical linearization scheme and relies on the determination of an equivalent linear system replacing the original one in a specified sense. Further, the paper has developed a BEM – Newmark based numerical algorithm for determining the nonlinear response. The numerical results have shown that the proposed approximate approach provides a quite good estimate of the response statistics, and captures well the spectral content of the response. The quality of the approximation is irrespective of the fractional derivative order.

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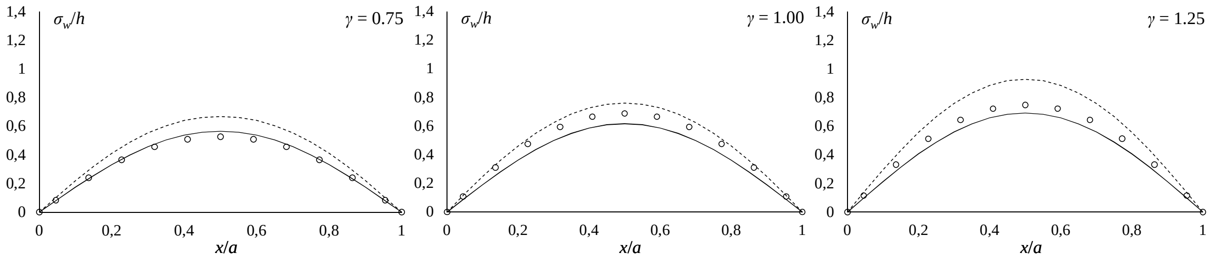

Figure 1. Standard deviation of the vertical plate displacement along the plate width considering various fractional derivative orders γ. Monte Carlo data (circles); statistical linearization solution (continuous line); linear solution (dotted line).

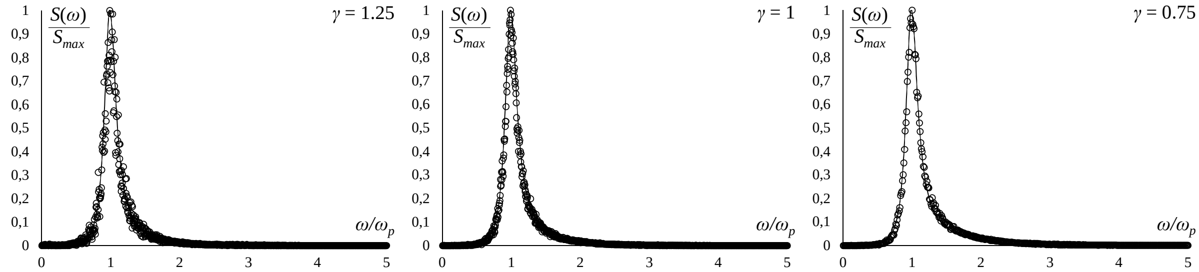

Figure 2. Power spectral density function of the vertical plate displacement calculated at the centre of the plate considering various fractional derivative orders γ. Monte Carlo data (circles); statistical linearization solution (continuous line).

References

[1] I. Podlubny, *Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution and Some of Their Applications*, Academic Press, San Diego, CA, USA, (1998).

[2] P.G. Nutting, "A new general law of deformation", *Journal of The Franklin Institute-engineering and Applied Mathematics*, **191**, 679-685 (1921).

[3] A. Gemant, "A method of analyzing experimental results obtained from elasto-viscous bodies", *Physics*, **7**, 311-317 (1936).

[4] R.L. Bagley and P.J. Torvik, "A Theoretical Basis for the Application of Fractional Calculus to Viscoelasticity", *Journal of Rheology*, **27**, 201-210 (1983).

[5] Y.A. Rossikhin and M.V. Shitikova, "Application of Fractional Calculus for Dynamic Problems of Solid Mechanics: Novel Trends and Recent Results ", *Applied Mechanics Reviews*, **63**, (2010).

[6] A.K. Grünwald, "Ueber "begrenzte" derivationem und deren anwendung", *Zeitschreft f. Mathematik u. Physik*, **12**, 441-480 (1867).

[7] J.B. Roberts and P.D. Spanos, *Random Vibration and Statistical Linearization*, Dover Publications, Mineola, New York, USA, (2003).

[8] J.T. Katsikadelis and M.S. Nerantzaki, "Non-linear analysis of plates by the Analog Equation Method", *Computational Mechanics*, **14**, 154-164 (1994).

[9] P.D. Spanos and G.I. Evangelatos, "Response of a non-linear system with restoring forces governed by fractional derivatives-Time domain simulation and statistical linearization solution", *Soil Dynamics and Earthquake Engineering*, **30**, 811-821 (2010).