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SEISMIC BEHAVIOUR OF STRUCTURES WITH PLASTIC SHEAR EFFECTS

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Abstract. *The dynamic response of building structures subjected to seismic loads has been often examined using the single-degree-of-freedom model, that provides a good estimation of the fundamental response mode, which is normally responsible for overall structural failure. A SDOF analysis can give a preliminary assessment for a protective structure, even in cases in which the constitutive models are somewhat more complex. This paper presents a general treatment to develop approximate solutions for rigid-plastic response of SDOF structures subjected to base harmonic pulse by means of a numerical procedure on purpose developed. The failure is assumed depending on the formation of a single shear hinge and the results are expressed in general terms for application to real cases.*

Sommario. *La risposta dinamica degli edifici alti soggetti ad azioni sismiche è stata spesso, in via semplificata, esaminata con riferimento ai sistemi ad un grado di libertà, che forniscono informazioni importanti sul primo modo di vibrare della struttura e quindi sulle modalità di collasso. In questo lavoro viene presentata una trattazione generale che deriva soluzioni approssimate per la risposta rigido plastica di strutture ad un grado di libertà soggette a pulsazioni armoniche. Si presentano i risultati di una procedura numerica sviluppata ad hoc e relativi a una mensola verticale in cui il collasso avviene per formazione di una cerniera a taglio all'incastro.*

1 INTRODUCTION

Rigid plastic approaches have been used both for steel and reinforced concrete structures, due to the advantages offered in term of simplified design and seismic assessment procedures[1]. A large amount of literature now exists for the dynamic plastic bending response of structural elements, since bending and shearing hinges represent general characteristics of the dynamic plastic response of several one and two-dimensional structural elements under transverse load[2]. In both the bending and shear problems the important question is linked to the localization and extension of plastic hinges[3]. In fact in some cases the length of the plastic hinge zone is evaluated by numerical experiments for a basic response pattern and taken into account by means of a phenomenological factor. In the case of structures in which shear failure can be easily recognized (rc shear walls, elevator shafts, whole framed structure buildings) the continuum model provide useful results. In some cases it has been shown that even when only flexural behaviour has been taken into account, a different shear distribution can be evaluated. This paper presents a general treatment to develop approximate solutions for rigid-plastic response of structures subjected to base harmonic pulse, that has been shown in literature as an appropriate approach to the dynamic analysis. A numerical procedure has been on purpose developed. The constitutive model involved is the rigid plastic one and the fundamental equation of the problem[4] are presented. The case study is a vertical cantilever beam, with base support, constant mass and inertia distribution, subjected to only ground acceleration. The procedure can be successfully applied to the case of impulse loading[5]. The natural development of the problem takes into account the presence of a stochastic forcing load on the system[6].

2 STATEMENT OF THE PROBLEM

Reference is made to an elastic perfectly plastic body Ω where $\partial\Omega = \partial\Omega_t \bigcup \partial\Omega_u$ is the total boundary of the body, $\partial\Omega_t \bigcap \partial\Omega_u = \varnothing$, $\partial\Omega_t$, $\partial\Omega_u$ are the free and the constrained boundary of the body Ω, $\lambda(t)\,\mathbf{t}(\mathbf{x})$ are the surface loads on the free boundary, $\partial\Omega_t$ $\lambda(t)\,\mathbf{b}(\mathbf{x})$, are the body forces in Ω, $\dot{\mathbf{u}}_g(\mathbf{x},t)$ is the assigned velocity vector and $\dot{\mathbf{u}}(\mathbf{x},0)=\mathbf{0}$ on $\partial\Omega_u$, $\lambda(t)$ is the time dependent load multiplier function. The problem solution is defined by the velocity, strain rate and stress fields respectively denoted with $\dot{\mathbf{u}}(\mathbf{x},t)$, $\dot{\boldsymbol{\varepsilon}}(\mathbf{x},t)$, $\boldsymbol{\sigma}(\mathbf{x},t)$. The strain rate and velocity fields satisfy the admissibility conditions, while the stress field is equilibrated with the applied loads. The approximate response field[4] is assumed in the form, where $\boldsymbol{\Phi}(\mathbf{x})$ is an assigned vector i.e. the modal form of the motion, depending on the initial position only and L(t) is an unknown scalar function of the time[5]. The approximate solution is composed by the admissible velocity field $\dot{\mathbf{u}}^*(\mathbf{x},t)$ satisfying the boundary condition $\dot{\mathbf{u}}^*(\mathbf{x},t)=\dot{\mathbf{u}}_g(\mathbf{x},t)$ on $\partial\Omega_u$ and the initial conditions $\dot{\mathbf{u}}^*(\mathbf{x},0)=\mathbf{0}$ in Ω and $\partial\Omega_t$. The stress field $\boldsymbol{\sigma}(\mathbf{x},t)$ is in balance with the assigned loads $\lambda(t)\,\mathbf{t}(\mathbf{x})$ and $\lambda(t)\,\mathbf{b}(\mathbf{x})$ and with the inertial forces $-\mu(\mathbf{x})\ddot{\mathbf{u}}^*(\mathbf{x},t)$, being $\mu(\mathbf{x})$ the mass density function. Applying the principle of virtual power:

$$\int_\Omega \mu(\ddot{\mathbf{u}}-\ddot{\mathbf{u}}^*)\cdot(\dot{\mathbf{u}}-\dot{\mathbf{u}}^*)\,d\Omega + \int_\Omega (\boldsymbol{\sigma}-\boldsymbol{\sigma}^*)\cdot(\dot{\boldsymbol{\varepsilon}}-\dot{\boldsymbol{\varepsilon}}^*)\,d\Omega = 0. \tag{1}$$

A suitable form of the first derivative of $\Delta(t)=\frac{1}{2}\int_\Omega \mu(\dot{\mathbf{u}}-\dot{\mathbf{u}}^*)\cdot(\dot{\mathbf{u}}-\dot{\mathbf{u}}^*)\,d\Omega$ is calculated after some manipulation as follows:

$$\frac{d\Delta}{dt} = -\int_\Omega (\boldsymbol{\sigma}-\boldsymbol{\sigma}^*)\cdot\boldsymbol{\varepsilon}^* + (\boldsymbol{\sigma}^*-\boldsymbol{\sigma})\cdot\dot{\boldsymbol{\varepsilon}}^* d\Omega = \int_\Omega (\boldsymbol{\sigma}^*-\boldsymbol{\sigma})\cdot\dot{\boldsymbol{\varepsilon}} + (\boldsymbol{\sigma}-\boldsymbol{\sigma}^*)\cdot\dot{\boldsymbol{\varepsilon}}^* d\Omega\,. \tag{2}$$

Since the normality rule holds true the above relation becomes:

$$\frac{d\Delta}{dt}=\int_{\Omega}\left[(\boldsymbol{\sigma}^{*}-\boldsymbol{\sigma})\cdot\dot{\boldsymbol{\varepsilon}}+(\boldsymbol{\sigma}-\boldsymbol{\sigma}^{*})\cdot\dot{\boldsymbol{\varepsilon}}^{*}\right]d\Omega\le\int_{\Omega}(\boldsymbol{\sigma}^{*}-\boldsymbol{\sigma}^{*})\cdot\dot{\boldsymbol{\varepsilon}}^{*}d\Omega=\Gamma(t)\ge 0 \tag{3}$$

where the last term in (3) results non-negative. The approximation measure $\Delta(t)$ in (3) is obtained integrating the previous relation from 0 to time t:

$$\Delta(t)\le\Delta^{+}(t)=\int_{0}^{t}\Gamma(t)\,dt. \tag{4}$$

The real velocity field at time t has been substituted with the approximate one. Since $\dot{\mathbf{u}}^{*}(\mathbf{x},0)=0$ the function L(t) satisfies the initial condition L(0)=0. The acceleration field and the strain velocity field $\dot{\boldsymbol{\varepsilon}}^{*}$ can be determined, in the form:

$$\ddot{\mathbf{u}}^{*}(\mathbf{x},t)=\boldsymbol{\Phi}(\mathbf{x})\dot{L}(t)\quad;\quad\dot{\boldsymbol{\varepsilon}}^{*}(\mathbf{x},t)=\boldsymbol{\Psi}(\mathbf{x})L(t) \tag{5}$$

being $\boldsymbol{\Psi}(\mathbf{x})$ the vector involving the strain generalized components associated to modal vector $\boldsymbol{\Phi}(\mathbf{x})$. The principle of virtual velocity gives:

$$\int_{S_L}\mathbf{p}(\mathbf{x},t)\cdot\dot{\mathbf{u}}^{*}dS+\int_{\Omega}[\mathbf{F}(\mathbf{x},t)-\mu\ddot{\mathbf{u}}^{*}]\cdot\dot{\mathbf{u}}^{*}d\Omega=\int_{\Omega}\boldsymbol{\sigma}^{*}(\mathbf{x},t)\cdot\dot{\boldsymbol{\varepsilon}}^{*}(\mathbf{x},t)\,d\Omega \tag{6}$$

so that the L(t) value can be determined through integration:

$$\dot{L}(t)=\frac{\int_{S_L}\mathbf{p}^{T}(\mathbf{x},t)\boldsymbol{\Phi}(\mathbf{x})\,dS+\int_{\Omega}\mathbf{F}^{T}(\mathbf{x},t)\boldsymbol{\Phi}(\mathbf{x})\,d\Omega-\int_{\Omega}\boldsymbol{\sigma}^{*}(\mathbf{x},t)\boldsymbol{\Psi}(\mathbf{x})\,d\Omega}{\int_{\Omega}\mu(\mathbf{x})\boldsymbol{\Phi}^{T}(\mathbf{x})\boldsymbol{\Phi}(\mathbf{x})\,d\Omega}. \tag{7}$$

The method can be applied to pulse loads, with the two conditions that the tractions applied to $\partial\Omega_t$ are null and the initial velocities are prescribed over the whole structure at time $t=0$; thereafter, no external forces do work on the structure.

3 THE CANTILEVER BEAM

The dynamic response to base support excitation of a vertical rigid-perfectly-plastic cantilever beam is considered. The local yield kinematism corresponds to the activation of a shear hinge in which the total shear force T(z,t) attains its bound values. The mechanical characteristics are shown in Fig. 2, where $\mu(z)$ is the linear mass density of the beam, $u_g(t)$ is the horizontal motion of the supported section, T(z,t) is the shear stress whose bounds are $T_{0^+}(z)$ and $T_{0^-}(z)$, $\theta(z,t)$ is the shear strain and h is the total length of the beam. The plastic deformation depend on the shear stress only, so that reference is made to the shear strain as above defined. The structure response is then characterized by a rigid-plastic law depending on the shear strain rate $\dot{\theta}(z,t)$ according the following relations:

$$\left[T(z,t)-T_{o+}(z)\right]\left[T(z,t)+T_{o-}(z)\right]\dot{\theta}(z,t)=0\quad,\quad -T_{o-}(z)\le T(z,t)\le T_{o+}(z). \tag{8}$$

In the cross section at the z level the relative and absolute displacements are respectively:

$$v(z,t)=\int_{0}^{z}\theta(x,t)dx=\int_{0}^{z}\int_{0}^{t}\dot{\theta}(x,\tau)d\tau\,dx\quad,\quad u(z,t)=u_g(t)+v(z,t). \tag{9}$$

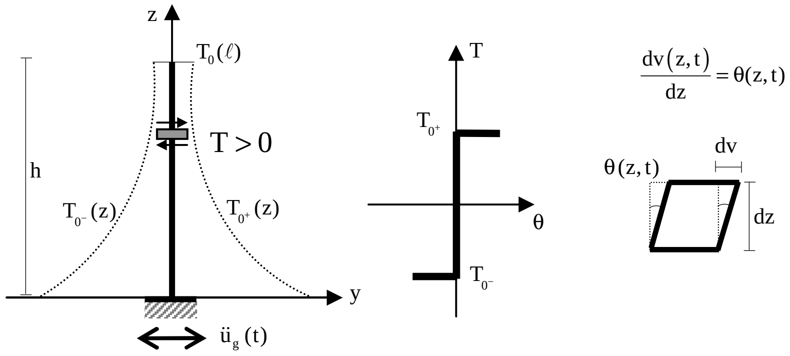

Figure 1. Rigid-plastic cantilever beam with base motion and limit shear values (a); constitutive law (b); shear strain of an infinitesimal element of the beam

From (9) the instantaneous condition of dynamic equilibrium involves the inertial forces only:

$$\frac{\partial T(z,t)}{\partial z}=\mu(z)\ddot{u}(z,t)=\mu(z)\left[\ddot{u}_g(t)+\ddot{v}(z,t)\right]. \tag{10}$$

The kinematic compatibility states that the plastic strain rate is equal to zero everywhere except in the active hinges:

$$\dot{\theta}(z,t)=\frac{\partial\,\dot{v}(z,t)}{\partial z}=\dot{v}'(z,t). \tag{11}$$

Integrating the above relation (11) with respect to *z*, it has:

$$\begin{aligned} v(z,t)&=\int_0^z v'(x,t)\,dx=\int_0^z \theta(x,t)\,dx=\int_0^h \theta(x,t)\,H(z-x)\,dx \\ \ddot{v}(z,t)&=\int_0^z \ddot{\theta}(x,t)\,dx=\int_0^h \ddot{\theta}(x,t)\,H(z-x)\,dx \end{aligned} \tag{12}$$

with H as the Heaviside function. In the following the case of only one shear hinge active is considered, whose time dependent position is $z_0(t)$. The relative displacement velocity and the plastic strain rate have the form:

$$\dot{v}(z,t)=\dot{v}_1(t)\,H[z-z_0(t)] \quad , \quad \dot{\theta}(z,t)=\dot{v}'(z,t)=\dot{v}_1(t)\;\delta[z-z_0(t)]$$

where $\delta[z-z_0(t)]$ is the Dirac function relative to the plastic hinge position. The expression of the shear stress in this case can be obtained:

$$T(z,t)=\ddot{u}_g(t)\int_h^z \mu(x)\,dx+\int_h^z \mu(x)\,\ddot{v}(x,t)dx \;\;,\;\; T(z,t)=\ddot{u}_g(t)\int_h^z \mu(x)\,dx+\ddot{v}_1(t)\int_h^z \mu(x)\,H[x-z_0(t)]dx$$

and $\mu(z)$ is the mass density. The mass functions are:

$$m(z)=\int_h^z \mu(x)\,dx \quad ; \quad m_H[z,z_o(t)]=\int_h^z \mu(x)\,H[x-z_o(t)]\,dx \tag{13}$$

with

$$\frac{dm(z)}{dz}=\mu(z) \quad , \quad \frac{\partial m_H[z,z_0(t)]}{\partial z}=\mu(z)\,H[z-z_0(t)] \quad , \quad \frac{\partial m_H[z,z_0(t)]}{\partial t}=-\mu(z)\,\dot{z}_o(t)H[z-z_0(t)]$$

T(z,t) can be written in function of the masses:

$$T(z,t)=\ddot{u}_g(t)\,m(z)+\ddot{v}_1(t)\,m_H[z,z_0(t)]$$

and the derivatives are:

$$\begin{aligned}
\frac{\partial T(z,t)}{\partial z}&=\mu(z)\,\ddot{u}_g(t)+\ddot{v}_1(t)\frac{\partial m_H[z,z_0(t)]}{\partial z}\\
\frac{\partial T(z,t)}{\partial t}&=m(z)\,\dddot{u}_g(t)+\dddot{v}_1(t)\,m_H[z,z_0(t)]-\dot{z}_o(t)\,\ddot{v}_1(t)\frac{\partial m_H[z,z_0(t)]}{\partial z}.
\end{aligned}$$

Considering that in the plastic hinge the shear is equal to yield value $T_0(z)$ and approximating the shear function by the Taylor series it has:

$$\begin{aligned}
T(z_0+dz_0,t+dt)=T_0(z_0)+&\left\{\mu(z_0)\,\ddot{u}_g(t)+\ddot{v}_1(t)\left.\frac{\partial m_H[z,z_0(t)]}{\partial z}\right|_{z=z_0}\right\}dz_0+\\
+&\left\{m(z_0)\,\dddot{u}_g(t)+\dddot{v}_1(t)\,m_H[z_0,z_0(t)]-\ddot{v}_1(t)\,\dot{z}_0(t)\left.\frac{\partial m_H[z,z_0(t)]}{\partial z}\right|_{z=z_0}\right\}dt.
\end{aligned}$$

If at the instant t+dt the position of the plastic shear hinge is z_o+dz_o, it must be also $T(z_0+dz_0,t+dt)=T_0(z_0+dz_0)$, and $T(z_0+dz_0,t+dt)=T_0(z_0)+T_0'(z_0)\,dz_0$ so that:

$$[\mu(z_0)\,\ddot{u}_g(t)-T_0'(z_0)]\;\dot{z}_o(t)+m(z_0)\,\dddot{u}_g(t)+\dddot{v}_1(t)\,m_H[z_0,z_0(t)]=0$$

whence, taking into account also the definition of masses, it is finally:

$$\dot{z}_0(t)=-\frac{m(z_0)\,\dddot{u}_g(t)+\dddot{v}_1(t)\,H(0)\,m[z_0(t)]}{\mu(z_0)\,\ddot{u}_g(t)-T_0'(z_0)}. \tag{14}$$

The quantity $\dot{z}_0(t)$ represents the evolution of the plastic shear hinge with the time. The motion of residual plastic deformations $\theta_r(z_0,t)$ is:

$$\theta_r(z_0,t)=\frac{\dot{v}_1(t)}{\dot{z}_0(t)}$$

and its value does not vary until the plastic hinge forms again at the same position. The derivation of the accelerations $\ddot{v}(z,t)$ is trivial.Given the function $\ddot{v}(z,t)$, it is possible to update all the quantities already known at the same time so that:

$$T(z,t+dt)=\int_z^h \mu(x)[\ddot{u}_g(t+dt)+v(x,t+dt)]\,dx.$$

The load due to the inertial forces at the abscissa z is given by:

$$q(z) = -\mu(z)\left[\ddot{u}_g(t) + \int_0^z \ddot{\theta}(x,t)\,dx\right]$$

and the total shear at the abscissa z is:

$$T(z,t) = -\int_h^z q(x)\,dx = \ddot{u}_g(t)\,m(z) + \int_h^z \mu(x)\int_0^x \ddot{\theta}(y,t)\,dy\,dx. \tag{15}$$

Remembering (8) the double condition of a cross section at the abscissa z_1 in active plastic phase and all the upper portion of the beam plastically inactive gives:

$$\begin{array}{l} [T(z_1,t) - T_{0^+}(z_1)][T(z_1,t) + T_{0^-}(z_1)] = 0 \\ -T_{0^-}(z) < T(z,t) < T_{0^+}(z) \quad , \quad \forall z > z_1 \end{array} \tag{16}$$

and then:

$$\begin{array}{ll} \ddot{u}_g(t) + \int_0^{z_1} \ddot{\theta}(y,t)\,dy = \dfrac{T_{0^+}(z_1)}{m(z_1)} & \ddot{u}_g(t) + \int_0^{z_1} \ddot{\theta}(y,t)\,dy = \dfrac{T_{0^-}(z_1)}{m(z_1)} \\ \dfrac{T_{0^-}(z)}{m(z)} < \dfrac{T_{0^+}(z_1)}{m(z_1)} < \dfrac{T_{0^+}(z)}{m(z)} \quad , \quad \forall z > z_1 & \dfrac{T_{0^-}(z)}{m(z)} < \dfrac{T_{0^-}(z_1)}{m(z_1)} < \dfrac{T_{0^+}(z)}{m(z)} \quad , \quad \forall z > z_1 \end{array} \tag{17}$$

In general, pulses that occur during earthquakes have qualitative and quantitative characteristics that can adequately be approximated. Several strong ground motions contain in fact an acceleration pulse responsible for most of the inelastic deformation of structures. These considerations are the basis of the numerical analysis performed. The case study is the vertical cantilever beam subjected to harmonic base motion and symmetric yield response of the cross section in the motion plane[7]. Reference is made to figure 2. The governing equations of the dynamic elastoplastic problem, based on (10) and (12), are:

$$\begin{cases} \dfrac{\partial T}{\partial z} = \mu[\ddot{u}_g(t) + \ddot{v}(z,t)] = \mu\left[\ddot{u}_g(t) + \int_0^z \ddot{\theta}(x,t)\,dx\right] = \mu\left[\ddot{u}_g(t) + \int_0^h \ddot{\theta}(x,t)\,H(z-x)\,dx\right] \\ T(h,t) = 0 \qquad \forall\, t \in [0,T] \\ T(z_1,t) = T_0(z_1,t), \quad z_1 \quad \text{plastic shear hinge position,} \;\; T_{0^+}(z_1,t) = T_0(z_1,t) = -T_{0^-}(z_1,t) \end{cases} \tag{18}$$

From the numerical analyses successive extensions and contractions of the plastic front can be evaluated, according the forcing time history. In the following pictures numerical results are reported: figure 2 contains the mechanical characteristics during the reduction of the plastic front, while figure 3 reports the behaviour in a single pulse. The extension of the plastic boundary moves starting from the abscissa $z_1 = h/2$ and evolves until the plastic excursion stops, corresponding to the attainment of the plastic threshold: spreading and contractions of the plastic front and related elastic returns can be observed.

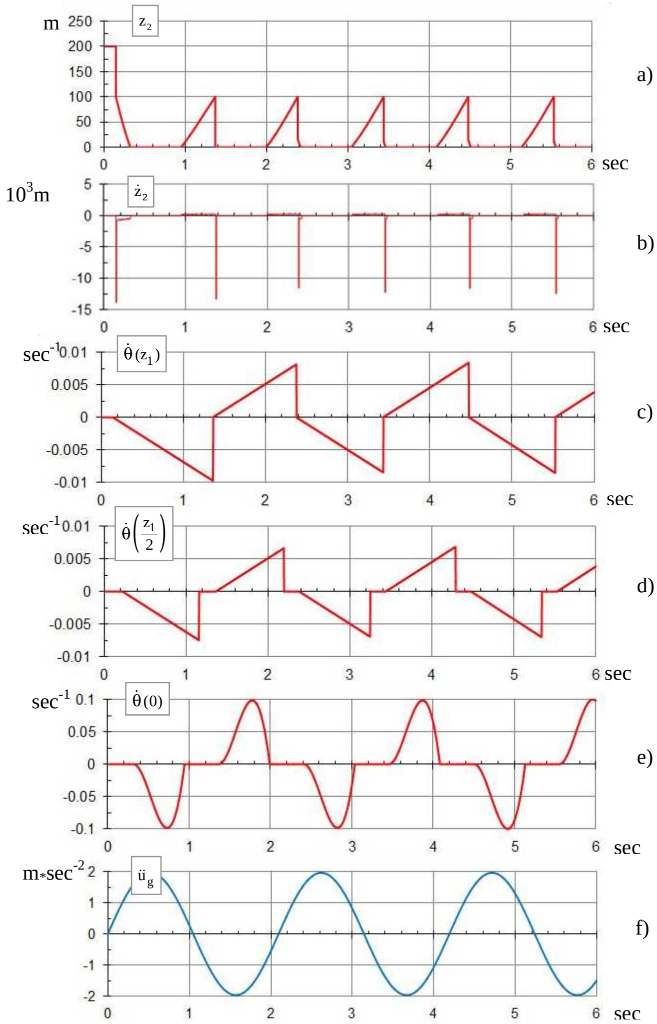

Figure 2.Time histories of the functions: lower bound (a) and velocity of the plastic front (b); plastic shear rate at the upper bound (c), in the middle (d) and at the base support (e); base accelerogram (f);

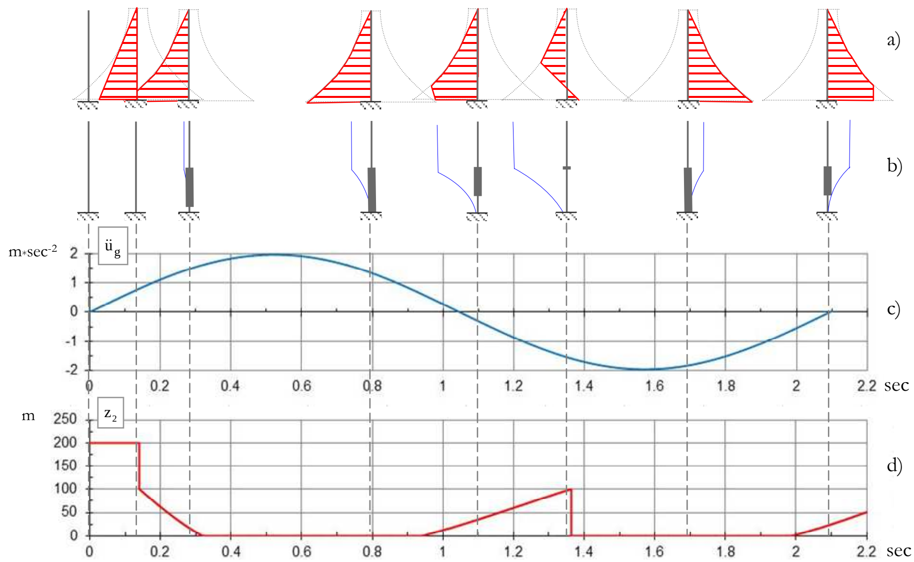

Figure 3. Shear stress (a) and deformed shape (b) versus the base accelerogram (c) and the lower bound of the plastic hinge (d)

The input data used in the numerical analysis are:

$$\mu(z) = 5\times10^4 \,\mathrm{kg\,m^{-1}} \quad , \quad \ddot{u}_g(t) = a_0 \sin\omega t \quad \text{with} \quad a_0 = 0.2\,g \quad , \quad \omega = 3\sec^{-1}$$

$$D = 6 \sec \quad , \quad T_0(z) = az^2 + bz + c \quad \text{with} \quad a = 2\times10^3 \,\mathrm{Nm^{-2}}, \quad b = 8\times10^5 \,\mathrm{Nm^{-1}}, \quad c = 10^8 \,\mathrm{N}$$

The behaviour is represented above, together with a schematic picture of the cantilever beam and its deformed shape corresponding to the actual plastic hinge extension. The maximum velocity lower bound corresponds to the attainment of the maximum spreading of the plastic front. The time histories corresponding to the evolution of the shear strain rate in three significant positions are picted. It is worth noting that both the diagrams in figure 3 have a sudden change of sign due to the inversion of the pulse sign.

4 CONCLUSIONS

A dynamic analysis method involving rigid-plastic behaviour has been presented. The response of a SDOF structure subjected to harmonic pulse base motion has been calculated in the whole time domain by means of a step by step integration procedure. The procedure presented is suitable when a shear failure can be recognized in the global behaviour. The procedure can be extended with a limited computational effort to elastoplastic structures with several degree of freedom and generic ground acceleration. The proposed procedure makes a set of necessary information available to the building designer: the extension of the plastic front to detect the localization of damage area and the structural elements of special attention are supplied by the analysis. The analysis of the system under stochastic loading force is undergoing in the next future.

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