UNIVERSITÀ DEGLI STUDI DI PALERMO

Meccanica dei Materiali e delle Strutture
Vol. VI (2016), no.1, pp. 155-162
ISSN: 2035-679X
Dipartimento di Ingegneria Civile, Ambientale, Aerospaziale, Dei Materiali
DICAM

THE DISTRIBUTION OF THE MAXIMUM WIND-INDUCED RESPONSE OF STRUCTURES

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Key words: Distribution of the maximum, Dynamic response, Monitoring network, Monte Carlo simulation, Stationary Gaussian process, Wind engineering.

Parole chiave: Distribuzione del massimo, Risposta dinamica, Rete di monitoraggio, Simulazione Monte Carlo, Processo stazionario Gaussiano, Ingegneria del vento.

Abstract. *The distribution of the maximum response of structures to wind actions is usually determined by Davenport's model assuming that the structural response is a random stationary Gaussian process and the up-crossings of high response thresholds are rare independent events. The hypothesis that wind actions are stationary limits analyses to synoptic phenomena. The hypothesis that wind actions are Gaussian implies that turbulence is small and atmospheric stratification is neutral. The hypothesis that the up-crossings of high response thresholds are rare independent events raises some doubts with regard to structures with low fundamental frequency and/or small damping coefficient. This paper aims at inspecting this topic with special regard to the properties of real wind velocity records and their influence on the distribution of the maximum response, the classical use of theoretical models and their reliability for flexible and low damped structures. Analyses are carried out in the classical framework of stationary Gaussian processes.*

Sommario. *La distribuzione del massimo della risposta di strutture soggette alle azioni del vento è solitamente studiata mediante il modello di Davenport assumendo che la risposta strutturale sia un processo aleatorio stazionario Gaussiano e che gli attraversamenti dal basso di soglie elevate della risposta siano eventi rari e indipendenti. L'ipotesi che le azioni del vento costituiscano un processo stazionario circoscrive l'analisi agli eventi sinottici. L'ipotesi che le azioni del vento costituiscano un processo Gaussiano implica piccola turbolenza e condizioni atmosferiche neutrali. L'ipotesi che gli attraversamenti dal basso di soglie elevate della risposta siano eventi rari e indipendenti pone alcuni dubbi per strutture con bassa frequenza fondamentale e/o limitato smorzamento. Questo lavoro si propone di analizzare questo tema con particolare attenzione alle proprietà dei segnali reali della velocità del vento e alla loro influenza sulla distribuzione del massimo della risposta, all'uso classico dei modelli teorici e alla loro affidabilità per strutture molto flessibili e poco smorzate. Le analisi sono condotte nell'ambito classico dei processi stazionari Gaussiani.*

1 INTRODUCTION

The distribution of the maximum value of a random stationary process has attracted extensive research since the mid-twentieth century[20,4,17,3,18,11]. In this framework the model introduced by Davenport[2] with regard to the maximum response of structures subjected to aerodynamic wind loads represents a turning point not only for wind engineering. It presumes that structural response is a random stationary Gaussian process and the up-crossings of high response thresholds are rare independent events. The hypothesis that wind actions are stationary circumscribes analyses to synoptic phenomena. The hypothesis that wind actions are Gaussian implies that atmospheric turbulence is small, so the quadratic term of fluctuations is negligible, and neutral atmospheric conditions occur[16]. The hypothesis that the up-crossings of high response thresholds are rare independent events raises some doubts with regard to structures with low fundamental frequency and/or small damping.

The quadratic term of turbulence attracted huge research in several fields of stochastic mechanics[2,21]. Studies carried out on the wind-excited response of structures show its limited role with reference to the mean value of the maximum response, and increasing importance with regard to the tail of its distribution.

The distribution of the maximum response for not rare and independent threshold up-crossings received wide interest in process theory and stochastic mechanics[5,1]. However, few papers in wind engineering studied the occurrence of up-crossing in clumps[19,6]; this is mainly due to the "long" period on which the maximum is evaluated (between 10 minutes and 1 hour, e.g. Van der Hoven[11]) and, even more, to considering Davenport's model as a sort of "axiom".

This paper aims at inspecting the distribution of the maximum response with special regard towards some topics almost ignored in literature: the properties of real wind velocity records and their influence on the maximum distribution, the classical use of theoretical models and their reliability for structures with low fundamental frequency and/or small damping ratio. Analyses are carried out in the classical framework of stationary Gaussian processes, without examining the widely debated role of the quadratic term of turbulence.

Section 2 formulates the equation of motion of a point-like Single-Degree-Of-Freedom (SDOF) system subjected to aerodynamic wind loads. Section 3 investigates the distribution of the maximum response to 205 real wind velocity records detected by the monitoring network of the "Wind and Ports" (WP)[7] and "Wind, Ports and Sea" (WPS)[12] European Projects. Section 4 studies the distribution of the maximum response to 10,000 simulated wind velocity records, whose target power spectral density (psd) is equal to the mean psd of the real records. Section 5 expresses the distribution of the maximum response through Davenport[3], Vanmarcke[5] and Der Kiureghian[1] models, comparing the results with those obtained by Monte Carlo simulations. Section 6 summarizes the main conclusions and provides some prospects.

2 EQUATIONS OF MOTION

Let us consider an ideal point-like SDOF linear system subjected to the alongwind aerodynamic force $f(t)=\rho v^2(t)Ac_D/2$, where ρ is the air density, A is the area of the exposed surface, c_D is the drag coefficient, $v(t)=\bar{v}+v'(t)$ is the wind velocity, $\bar{v}$ and v' being, respectively, the mobile mean wind velocity over ΔT and the residual turbulent fluctuation. The alongwind displacement x is given by the solution of the differential equation of motion:

$$\ddot{x}(t)+2\xi(2\pi n_0)\dot{x}(t)+(2\pi n_0)^2 x(t)=\frac{1}{m}f(t) \qquad (1)$$

in which $t \in [0, \Delta T]$ is the time, ΔT = 10 minutes, n_0 is the fundamental frequency, ξ is the damping coefficient, m is the mass. Using the above definitions of $f(t)$ and $v(t)$, Eq.1 becomes:

$$\ddot{x}(t) + 2\xi(2\pi n_0)\dot{x}(t) + (2\pi n_0)^2 x(t) = \frac{1}{2m}\rho\bar{v}^2 A c_D \left[1 + 2I_v\tilde{v}'(t) + I_v^2\tilde{v}'^2(t)\right] \tag{2}$$

where $I_v = \sigma_v/\bar{v}$ is the turbulence intensity and $\tilde{v}'(t) = v'(t)/\sigma_v$ is the reduced turbulent fluctuation, σ_v being the standard deviation (std) of v.

Let us introduce the reduced (dimensionless) displacement $d(t) = x(t)/\bar{x}$, the reduced fundamental frequency $\tilde{n}_0 = n_0 h/\bar{v}$ and the reduced time $\tilde{t} = t\bar{v}/h$, where $\bar{x}$ is the mean static displacement and h is the reference height above ground at which the wind velocity is assigned. Accordingly, Eq. 2 may be rewritten in the non-dimensional form:

$$\ddot{d}(\tilde{t}) + 2\xi(2\pi\tilde{n}_0)\dot{d}(\tilde{t}) + (2\pi\tilde{n}_0)^2 d(\tilde{t}) = (2\pi\tilde{n}_0)^2\left[1 + 2I_v\tilde{v}'(\tilde{t}) + I_v^2\tilde{v}'^2(\tilde{t})\right] \tag{3}$$

Eq. 3 points out that d depends on $\tilde{v}'$ and the three non-dimensional parameters $\tilde{n}_0$, ξ, and I_v. It is worth noting, however, the key role of the spectral representation of $\tilde{v}'$. In particular, expressing the psd of $\tilde{v}'$, S_v, as a function of the reduced frequency $\tilde{n} = nh/\bar{v}$, d does not depend on any other parameter; instead, expressing its psd as a function of $\tilde{f} = nL_v/\bar{v}$, L_v being the integral length scale of $\tilde{v}'$, d depends also on L_v/h.

Finally, let us define $S_d = |d|_{max}$ as the maximum value of the d modulus. By analogy with Solari[9], this quantity is also referred to as the synoptic wind response spectrum.

3 MAXIMUM RESPONSE TO REAL VELOCITY RECORDS

WP[7] and WPS[12] are two European Projects carried out between 2009 and 2015 to handle the wind forecast in the port areas of Genoa, Savona, Vado Ligure, La Spezia, Livorno, Bastia and L'Île-Rousse through an integrated system made up of an extensive in-situ monitoring network, the numerical simulation of wind fields, the statistical analysis of the wind climate, and algorithms for medium- (1-3 days) and short- term (0.5-2 hours) forecasting. The monitoring network is made up of 28 ultrasonic anemometers that detect the wind speed and direction with a precision of 0.01 m/s and 1 deg, respectively, 3 LiDARs (LIght Detection And Ranging) and 3 weather stations, each one including an ultrasonic anemometer. The height of the anemometers varies from 10 m to 84 m above ground level. The sampling rate is 10 Hz, with the exception of the sensors in the Ports of Bastia and L'Île-Rousse whose sampling rate is 2 Hz.

A set of local servers in each Port Authority receives the measured data, elaborates basic statistics and sends a central server in DICCA two files that contain, respectively, raw data and statistical estimates. The DICCA operational centre stores the data into a central dataset by a codified procedure. Later on, a semi-automated method is applied[16] in order to extract and separate different intense wind events, i.e. extra-tropical cyclones, thunderstorm outflows and intermediate events. This paper analyses a representative set of records detected in extra-tropical cyclones.

To this aim, 205 10-min wind velocity records were selected with mean wind velocity $\bar{v} >$ 10 m/s assuring the occurrence of neutral atmospheric conditions. The mean value of the skewness and kurtosis of the whole set of records are, respectively, γ = -0.03 and κ = 2.86; this points out reasonably good Gaussian properties.

Fig. 1 shows two sample records including the 10-min mean value and the 30-s moving average: scheme (a) provides an excellent example of a stationary Gaussian event, while scheme (b) shows the presence of a wave with period greater than 10 minutes. The possible occurrence of synoptic signals with long period harmonic content makes questionable the stationarity property and the use of records whose length is ΔT = 10 minutes; this also confirms some critical issues on the existence of the spectral gap or, at least, some of its properties widely shared in literature[15].

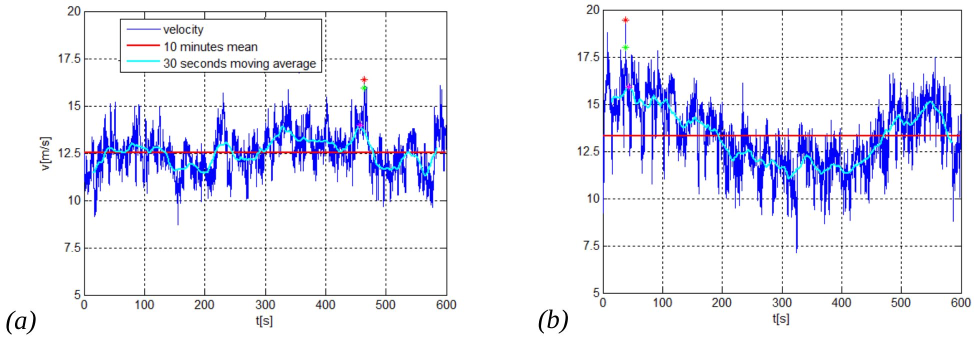

Figure 1: Real velocity records detected by the WP and WPS monitoring network: (a) anemometer 2 of the Port of Genoa, 24/04/2012 from 15:30; (b) anemometer 3 of the Port of Livorno, 07/12/2011 from 17:20.

The spectral analysis of the dataset points out, as expected, very wide variability. However, the mean value of S_v closely matches the formula:

$$\frac{nS_v(n)}{\sigma_v^2}=\frac{\tilde{n}/\tilde{n}_m}{\left(1+1.5\tilde{n}/\tilde{n}_m\right)^{5/3}} \tag{4}$$

where $\tilde{n}_m = 0.039$. The preliminary use of Eq. 4 has the advantage of avoiding that the dynamic response depends on L_v/h. However, the role of L_v is still matter of evaluations.

The analysis of I_v points out wide variability depending on the sensor height and the wind direction. Thus, the set of 205 records has been divided into 3 sub-data sets whose properties are shown in Table 1; it reports, for each sub-set, the range of I_v, the number N of the records that belong to the sub-set, and the mean turbulence intensity $\langle I_v \rangle$. No relevant difference has been observed among these sub-sets with reference to Eq. 4.

I_v range	N	$\langle I_v \rangle$
$I_v < 0.15$	84	0.12
$0.15 < I_v < 0.20$	75	0.17
$I_v > 0.20$	46	0.23

Table 1: Main properties of the 3 sub-datasets of real records characterized by different I_v values.

The evaluation of the dynamic response of the SDOF system described in Section 2 and its maximum value have been carried out neglecting the quadratic term of turbulence. The integration of the equation of motion has been carried out in the state space introducing a 1-min long Hamming windowing before the beginning of the 10-min period in which the maximum response has been calculated. Parametric analyses have been carried out in order to cover the whole set of really existing structural types.

Fig. 2(a) shows the mean value of S_d as a function of $\tilde{n}_0$ for $I_v \in$ (0.15-0.20), on varying ξ. Fig. 2(b) shows the spread of S_d around its mean value for ξ = 0.002. In addition it shows the histograms of S_d as compared with a reference I type distribution (with the same mean and std) for $\tilde{n}_0$ = 1 and 5. On increasing ξ, $\langle S_d \rangle$ and its spread reduce. On increasing I_v, $\langle S_d \rangle$ increases.

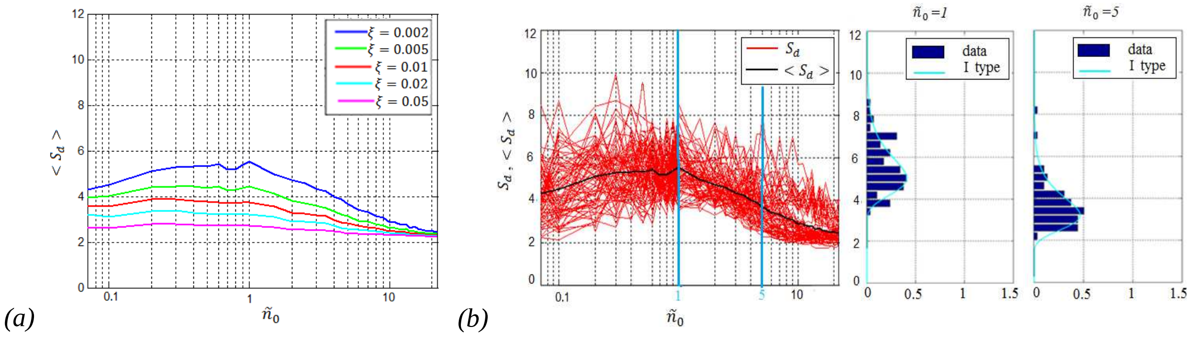

Figure 2: Mean value (a) and spread (b) of S_d to real velocity records: $Iv \in$ (0.15-0.20); (b) ξ = 0.002.

4 MAXIMUM RESPONSE TO SIMULATED VELOCITY RECORDS

To investigate the distribution of the maximum response to simulated velocity records, 10,000 time-histories of the reduced turbulence intensity $\tilde{v}'$ have been generated through a Monte Carlo procedure based upon the random phase spectral method[13,14].

Coherently with the properties of the real velocity records, the simulations were carried out with a time step Δt = 0.1 s on a time interval ΔT = 10 minutes. The target psd defined by Eq. 4 was adopted and its harmonic content was simulated between 0 and the cut-off frequency n_c = 5 Hz. This frequency range was sub-divided into 10,000 sub-ranges Δn =5x10^{-4} Hz wide.

The mean values of the skewness and kurtosis of this set of records are, respectively, γ = 0.00 and κ = 2.94; this points out almost perfect Gaussian properties.

Likewise Fig. 1(a), and differently from Fig. 1(b), no simulated wind velocity record contains wave components with period greater than 10 minutes.

The evaluation of the dynamic response of the ideal SDOF system described in Section 2 and its maximum value have been carried out using the same approach illustrated in Section 3.

Analogously to Fig. 2(a), Fig. 3(a) shows the mean value of S_d as a function of $\tilde{n}_0$ for I_v = 0.17 on varying ξ. The comparison of Fig. 2(a) and Fig. 3(a) points out a good agreement.

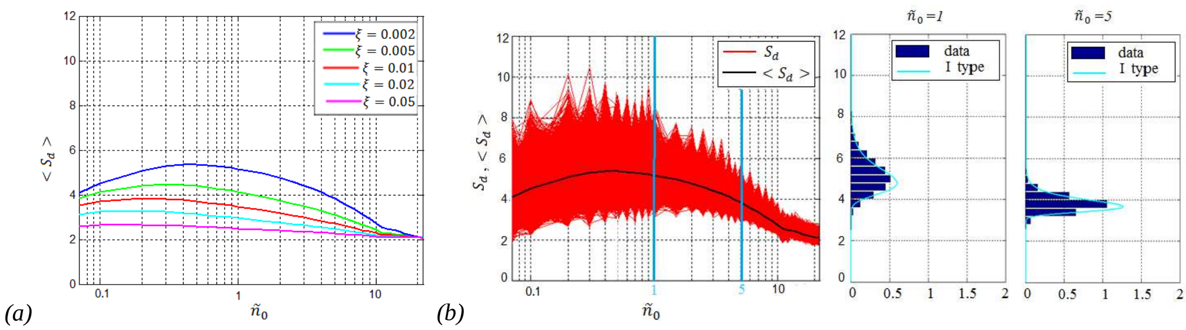

Figure 3: Mean value (a) and spread (b) of S_d to simulated velocity records: Iv = 0.17; (b) ξ = 0.002.

Analogously to Fig. 2(b), Fig. 3(b) shows the spread of S_d around its mean value for ξ = 0.002. In addition it shows the histograms of S_d compared with a reference I type distribution for $\tilde{n}_0$ =1 and 5. The comparison of Fig. 2(b) and Fig. 3(b) points out a similar qualitative trend. However, passing from real to simulated velocity records the spread of S_d considerably reduces. Some preliminary checks showed that this depends only marginally by the fact that real records are characterized by $I_v \in$ (0.15-0.20) while simulated ones imply I_v =0.17. Instead, it is apparent that the major spread of S_d for real records is mainly due to some records involving long wave periods; in any case, such a spread makes quite questionable the classic representativeness of the maximum response through its mean value.

5 MAXIMUM RESPONSE THROUGH THEORETICAL MODELS

The study of the theoretical expressions of the distribution of the maximum response is limited here to the most well-known and shared models formulated by Davenport (1964), Vanmarcke (1975) and Der Kiureghian (1981).

Davenport[3] developed closed form expressions of the pdf, mean value and std of the maximum response assuming the up-crossings of high response thresholds as rare independent Poissonian events. Accordingly, all the above quantities depend on σ and $\nu\Delta T$, σ and ν being, respectively, the std and the expected frequency of the response process.

Vanmarcke[5] generalised the formulation to threshold up-crossings in clumps. So he derived a new pdf of the maximum response that depends, besides σ and $\nu\Delta T$, on the spectral bandwidth parameter q. His paper, however, did not provide any explicit expression for the mean value and std of the maximum response; thus, these quantities have to be evaluated numerically.

Starting from Vanmarcke model, Der Kiureghian[1] derived simple semi-empirical formulae aiming to provide the mean value and std of the maximum response. They depend on σ, $\nu_e\Delta T$ and q, ν_e being the equivalent expected frequency.

The three models described above are applied herein determining their model parameters in the frequency-domain, as usual. Davenport and Vanmarcke models tend to coincide for large $\tilde{n}_0$ and/or ξ values, when threshold up-crossings are independent; for small $\tilde{n}_0$ and ξ values, when threshold up-crossings occur in clumps, Vanmarcke model substantially reduces the mean value of the maximum response. Der Kiureghian formulae provide a nearly perfect agreement with Vanmarke model except for the case of very small $\tilde{n}_0$ and ξ values, where it provides slight underestimations. Fig. 4(a) shows the mean value of S_d for ξ = 0.002, as evaluated by these three theoretical models and the time-domain simulations described in Section 4.

It is worth noting, as previously observed by Huang and Chen[5], the detachment between the theoretical and simulated mean values of the maximum response for small $\tilde{n}_0$ and ξ values (Fig. 4a). This difference is huge when using the classic Davenport method and only partially reduced by the Vanmarcke approach. This points out that for very small $\tilde{n}_0$ and ξ values the analyses should be carried out over longer time intervals in order to avoid transient effects.

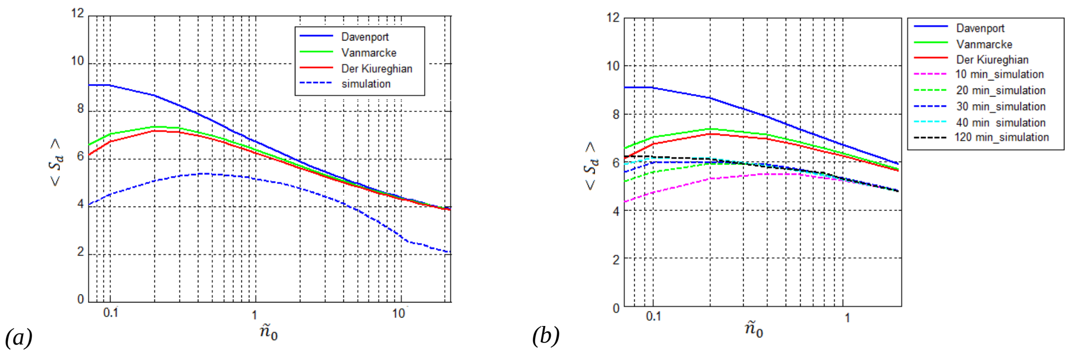

Figure 4: Mean value of the maximum response to simulated velocity histories as compared with some theoretical models: Iv = 0.17; ξ = 0.002.

Aiming to clarify this delicate issue, Fig. 4(b) focuses on the comparison of Fig. 4(a) by adding a set of diagrams related to Monte Carlo simulations and time-domain dynamic analyses carried out on time intervals 10, 20, 30, 60 and 120 minutes long. Frequency sub-ranges for simulations are reduced accordingly, and the simulation burden becomes extremely large if not prohibitive. The maximum value is evaluated in the last ΔT = 10-min period. Nevertheless, on increasing the time interval of the analyses, the results of the simulations tend to an asymptotic curve that remains well below the results provided by the Vanmarcke model. This raises a lot of questions on the huge errors involved by Davenport model, the limited improvement allowed by Vanmarcke model, the reliability of ΔT =10-min periods and the opportunity of using longer periods such as ΔT = 1 hour; this last fact, however, involves not only a high computational burden but also the relevant difficulty of finding reasonably stationary real wind velocity records.

6 CONCLUSION AND PROSPECTS

This paper investigates the distribution of the maximum structural response to synoptic wind actions with special regard, on the one hand, to the properties of real wind velocity records and their influence on the problem dealt with and, on the other hand, to the use of theoretical models, the representativeness of mean maximum response and their reliability for wind-sensitive structures.

The comparison between the maximum response of structures to real and simulated wind velocity records shows that the spread of the maximum response to real records is definitely higher than that to simulated ones. This seems mainly due to the existence of synoptic signals with long period harmonic contents exceeding ΔT =10 minutes. This remark makes quite questionable the property of stationarity and the analysis of records ΔT = 10-min long; it also confirms some critical issues on the existence of the spectral gap or at least on some of its properties widely shared in literature. In any case the increasing spread of the maximum response to real records makes quite questionable the identification of the maximum response with its mean value.

The comparison between the maximum response of structures to simulated wind velocity records and those evaluated through theoretical models shows impressive differences for low fundamental frequency and small damping values. Such differences are huge when using Davenport model and reduce only partially using the Vanmarcke approach and making recourse to Monte Carlo simulations and time-domain dynamic analyses that involve time intervals longer than ΔT =10 minutes. This fact raises several questions on the reliability of ΔT = 10-min periods and the opportunity of using longer periods such as ΔT =1 hour. Nevertheless, the use of long reference periods implies an extremely high numerical burden and relevant difficulties in identifying reasonably stationary real velocity records.

Acknowledgements: This research has been developed in the framework of the Project “Wind monitoring, simulation and forecasting for the smart management and safety of port, urban and territorial systems”, financed by Compagnia di San Paolo. The data exploited has been recorded by the monitoring network of the projects “Wind and Ports” and “Wind, Ports and Sea”, financed by European Territorial Cooperation Objective, Cross-border program Italy-France Maritime 2007-2013.

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