UNIVERSITÀ DEGLI STUDI DI PALERMO

Meccanica dei Materiali e delle Strutture
Vol. VI (2016), no.1, pp. 147-154
ISSN: 2035-679X
Dipartimento di Ingegneria Civile, Ambientale, Aerospaziale, Dei Materiali
DICAM

TIME- FREQUENCY VARYING RESPONSE FUNCTION OF NON-CLASSICALLY DAMPED LINEAR STRUCTURES UNDER SPECTRUM COMPATIBLE FULLY NON-STATIONARY STOCHASTIC EXCITATIONS

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Key words: Stochastic analysis; Random excitations; Structural safety; Non-geometric spectral moments.

Abstract. *The probabilistic analysis of structural systems subjected to seismic excitations requires the spectral characterization of both the input excitation and the structural response. Moreover, in order to reproduce the typical characteristics of real earthquakes ground-motion time history, the seismic excitation should be modelled as a fully non-stationary stochastic process. In the framework of stochastic structural analysis, the time-frequency varying response function (TFR) plays a central role in the evaluation of the statistics of the response of linear structural systems subjected to non-separable stochastic excitations.*
In this paper a method to evaluate in explicit closed-form TFR functions of linear non-classically damped structural systems under fully non-stationary excitation is proposed.

1 INTRODUCTION

One of the most important problem in seismic engineering is the correct characterization of the ground motion acceleration. In code-based seismic design and assessment it is often allowed the use of real records as an input for nonlinear dynamic analysis but, due to the difficulty in rationally relating the ground motions to the hazard at the site and the required selection criteria, the use of artificial spectrum-compatible accelerograms is preferred. Usually the stationary Gaussian stochastic model is adopted although the corresponding artificial accelerograms have an excessive number of cycles of strong motion and consequently they possess unreasonably high energy content[1]. Furthermore, the stationary model suffers the major drawback of neglecting the non-stationary characteristics of the real records: the changes in amplitude and frequency content. Indeed, the time-varying amplitude is due to the natural evolution of the earthquake ground motion, while the time-varying frequency content is prevalently due to different arrival times of the primary, secondary and surface waves that propagate at different velocities through the earth crust. The stochastic processes involving both the amplitude and the frequency changes are referred in literature as *fully non-stationary* random processes while the so-called *quasi-stationary* (or *uniformly* *modulated*) random processes present changes in amplitude only. In order to define the *fully non-stationary stochastic input*, the Priestley[2] spectral representation of non-stationary processes is the most adopted. In the Priestley model the *Evolutionary Power Spectral Density* (*EPSD*) function is introduced[2].

In the framework of stochastic structural analysis, the *time-frequency varying response* (*TFR*) function plays a central role in the evaluation of the statistics of the response of linear structural systems subjected to fully non-stationary stochastic excitations[3-7].

In the proposed approach, the main steps for the evaluation of the statistics of the response are: i) the evaluation of complex eigenproperties of the non-classically damped system; ii) the use of the state-variables to evaluate in explicit closed-form the *TFR* vector function of the structural response; iii) the evaluation in closed form of the *EPSD* matrix function of the response; iv) the definition of the *non-geometrical spectral moments* (*NGSM*s), in the time domain, as element of the pre-envelope covariance matrix[4,8]; v) the validation of the proposed procedure by the Monte Carlo Simulation.

2 EQUATIONS OF MOTION

Let consider the equation of motion of a linear quiescent n-degree-of-freedom (n-DOF) non-classically damped structural system whose dynamic behavior is governed by the following equation of motion:

$$\mathbf{M}\,\ddot{\mathbf{u}}(t)+\mathbf{C}\dot{\mathbf{u}}(t)+\mathbf{K}\mathbf{u}(t)=-\mathbf{M}\,\boldsymbol{\tau}F(t) \tag{1}$$

where **M**, **C**, and **K** are the $(n\times n)$ mass, damping, and stiffness matrices of the structure; $\mathbf{u}(t)$ is the $(n\times 1)$ vector of displacements, having for i-th element $u_i(t)$ and a dot over a variable denotes differentiation with respect to time; $\boldsymbol{\tau}$ is the influence vector and $F(t)$ is the ground acceleration. It has been recognized that, from a computational point of view, to operate in the modal subspace is more convenient than in the nodal space, for non-classically damped structures too. To this aim let introduce the modal coordinate transformation $\mathbf{u}(t)=\boldsymbol{\Phi}\,\mathbf{q}(t)$, where $\boldsymbol{\Phi}=\begin{bmatrix}\phi_1 & \phi_2 & \dots & \phi_m\end{bmatrix}$ is the modal matrix, of order $n\times m$, collecting the m eigenvectors ϕ_j, normalized with respect to the mass matrix $\mathbf{M}$, solutions of the eigenproblem $\mathbf{K}^{-1}\mathbf{M}\;\boldsymbol{\Phi}=\boldsymbol{\Phi}\;\boldsymbol{\Omega}^{-2}$ with the orthogonality condition $\boldsymbol{\Phi}^T\mathbf{M}\;\boldsymbol{\Phi}=\mathbf{I}_m$. The diagonal matrix $\boldsymbol{\Omega}$ collects the undamped natural circular frequency ω_j and $\mathbf{I}_m$ is the identity matrix of order m. Once the modal matrix $\boldsymbol{\Phi}$ is evaluated, by applying the coordinate transformations to Eq.(1), the following set of the first order differential equation can be written in the $2m$- state vector variable as[7]:

$$\dot{\mathbf{y}}(t)=\mathbf{D}\,\mathbf{y}(t)+\mathbf{w}F(t) \tag{2}$$

where

$$\mathbf{y}(t)=\begin{bmatrix}\mathbf{q}(t)\\ \dot{\mathbf{q}}(t)\end{bmatrix};\quad \mathbf{D}=\begin{bmatrix}\mathbf{0} & \mathbf{I}_m\\ -\boldsymbol{\Omega}^2 & -\boldsymbol{\Xi}\end{bmatrix};\quad \mathbf{w}=\begin{bmatrix}\mathbf{0}\\ \mathbf{p}\end{bmatrix} \tag{3}$$

In the latter equations $\mathbf{y}(t)$ is the vector of modal state variables; $\mathbf{p}=-\boldsymbol{\Phi}^T\mathbf{M}\,\boldsymbol{\tau}$ is the vector of participation factor and $\boldsymbol{\Xi}=\boldsymbol{\Phi}^T\mathbf{C}\;\boldsymbol{\Phi}$ is the generalized damping matrix which for non-classically damped systems is not a diagonal matrix. Therefore, the nodal response in state variables can be determined by the following back substitution:

$$\mathbf{z}(t)=\begin{bmatrix}\mathbf{u}(t)\\ \dot{\mathbf{u}}(t)\end{bmatrix}=\begin{bmatrix}\mathbf{\Phi} & \mathbf{0}\\ \mathbf{0} & \mathbf{\Phi}\end{bmatrix}\begin{bmatrix}\mathbf{q}(t)\\ \dot{\mathbf{q}}(t)\end{bmatrix}=\mathbf{\Pi}\,\mathbf{y}(t) \tag{4}$$

In order to decouple Eq.(2) a further coordinate transformation must be introduced, that is $\mathbf{y}(t)=\mathbf{\Psi}\,\mathbf{x}(t)$, where $\mathbf{\Psi}=\begin{bmatrix}\boldsymbol{\psi}_1 & \boldsymbol{\psi}_2 & \dots & \boldsymbol{\psi}_m & \boldsymbol{\psi}_1^* & \boldsymbol{\psi}_2^* & \dots & \boldsymbol{\psi}_m^*\end{bmatrix}$ is a complex matrix, of order $2m\times 2m$, collecting the $2m$ complex eigenvectors, $\boldsymbol{\psi}_j$, solutions of the eigenproblem $\mathbf{D}\,\mathbf{\Psi}=\mathbf{\Psi}\,\mathbf{\Gamma}$ with the orthogonality condition $\mathbf{\Psi}^T\mathbf{A}\,\mathbf{\Psi}=\mathbf{I}_{2m}$. The asterisk over a variable denotes complex conjugate quantity and $\mathbf{\Gamma}$ is a diagonal complex matrix listing the $2m$ eigenvalue of the non-classically damped system. Once the complex matrix $\mathbf{\Psi}$ is evaluated, by applying the new coordinate transformations to Eq.(2), the following set of $2m$ decoupled first order differential equations is obtained:

$$\dot{\mathbf{x}}(t)=\mathbf{\Gamma}\,\mathbf{x}(t)+\mathbf{\Psi}^T\mathbf{A}\;\mathbf{w}\,F(t);\quad \mathbf{x}(0)=\mathbf{0};\quad \mathbf{A}=\begin{bmatrix}\mathbf{\Xi} & \mathbf{I}_m\\ \mathbf{I}_m & \mathbf{0}\end{bmatrix} \tag{5}$$

where $\mathbf{x}(t)$ is complex vector of order $2m$. Finally the nodal response can be evaluated as:

$$\mathbf{z}(t)=\mathbf{\Pi}\,\mathbf{\Psi}\,\mathbf{x}(t) \tag{6}$$

3 CLOSED-FORM SOLUTION FOR THE TIME-FREQUENCY VARYING RESPONSE FUNCTION

The *TFR* function vector plays a central role in the evaluation of the statistics of the response of linear structural systems subjected to both separable or non-separable stochastic excitations. In fact, by means of this function, it is possible to evaluate in explicit form the *EPSD* function matrix of the response and consequently the non-geometric spectral moments, which are required in the prediction of the safety of structural systems subjected to non-stationary random excitations.

By means of the modal transformation Eq.(4), the one-sided *EPSD* function matrix of the nodal response, in state variables, can be evaluated, after very simple algebra, as follows:

$$\mathbf{G}_{\mathbf{zz}}(\omega,t_1,t_2)=\mathbf{\Pi}\mathbf{G}_{\mathbf{yy}}(\omega,t_1,t_2)\mathbf{\Pi}^T. \tag{7}$$

where the *EPSD* function matrix of the modal response, in state variables, $\mathbf{G}_{\mathbf{yy}}(\omega,t_1,t_2)$, is given as:

$$\mathbf{G}_{\mathbf{yy}}(\omega,t_1,t_2)=G_0(\omega)\,\mathbf{Y}^*(\omega,t_1)\,\mathbf{Y}^T(\omega,t_2) \tag{8}$$

with $G_0(\omega)$ is the embedded one-sided *power spectral density* (*PSD*) function of the stationary counterpart of the input process $F(t)$.

The function vector $\mathbf{Y}(\omega,t)$ is the so-called *TFR* function vector of the modal response, in state variables. The vector $\mathbf{Y}(\omega,t)$ can be evaluated as the solution of the following set of $2m$ first order differential equation[7]:

$$\dot{\mathbf{Y}}(\omega,t)=\mathbf{D}\,\mathbf{Y}(\omega,t)+\mathbf{w}\,f(\omega,t)\mathbb{U}(t); \quad \mathbf{Y}(\omega,\mathrm{t}_0)=\mathbf{Y}_0 \tag{9}$$

subjected to pseudo-force $f(\omega,t)=\exp(-\mathrm{i}\omega t)a(\omega,t)$. In the previous equation $\mathbb{U}(t)$ is the Unit Step function. Since the following coordinate transformation holds:

$$\mathbf{Y}(\omega,t)=\mathbf{\Psi}\,\mathbf{X}(\omega,t) \tag{10}$$

it follows that the first order differential equation (9) can be rewritten as a set of $2m$ decoupled firs order differential equations:

$$\dot{\mathbf{X}}(\omega,t)=\mathbf{\Gamma}\,\mathbf{X}(\omega,t)+\mathbf{\Psi}^T\mathbf{A}\;\mathbf{w}\,f(\omega,t)\mathbb{U}(t); \quad \mathbf{X}_0\equiv\mathbf{X}(\omega,t_0)=\mathbf{\Psi}^T\mathbf{A}\;\mathbf{Y}_0 \tag{11}$$

This equation represents the differential equation of motion, in state variable, of a quiescent dynamical system at time t=0, subjected to a pseudo-force $f(\omega,t)$. If the particular solution of Eq.(11), $\mathbf{X}_\mathrm{p}(\omega,t)$, can be determined in explicit form, the *TFR* vector function can be written as[9]:

$$\mathbf{X}(\omega,t)=\left\{\mathbf{X}_\mathrm{p}(\omega,t)+\exp(\mathbf{\Gamma}t)\left[\mathbf{X}_0-\mathbf{X}_\mathrm{p}(\omega,0)\right]\right\}\mathbb{U}(t) \tag{12}$$

Then, according to Eq.(10), the solution of Eq.(9) can be written as:

$$\begin{aligned}\mathbf{Y}(\omega,t)&=\mathbf{\Psi}\;\mathbf{X}(\omega,t)=\mathbf{\Psi}\left\{\mathbf{X}_\mathrm{p}(\omega,t)+\exp(\mathbf{\Gamma}t)\left[\mathbf{X}_0-\mathbf{X}_\mathrm{p}(\omega,0)\right]\right\}\mathbb{U}(t)\\&=\left\{\mathbf{Y}_\mathrm{p}(\omega,t)-\mathbf{\Theta}(t)\left[\mathbf{Y}_0-\mathbf{Y}_\mathrm{p}(\omega,0)\right]\right\}\mathbb{U}(t)\end{aligned} \tag{13}$$

where $\mathbf{\Theta}(t)$ is the transition matrix in modal subspace defined as:

$$\mathbf{\Theta}(t)=\mathbf{\Psi}\exp(\mathbf{\Gamma}t)\mathbf{\Psi}^T\mathbf{A} \tag{14}$$

The analytical expression of the particular solution vector $\mathbf{Y}_\mathrm{p}(\omega,t)=\mathbf{\Psi}\,\mathbf{X}_\mathrm{p}(\omega,t)$, can be easily obtained in closed form for the most common models of modulating function $a(\omega,t)$ proposed in literature[7].

In the framework of non-stationary analysis of structures, time-dependent parameters, very useful in describing the time-variant spectral properties of the stochastic process, are: i) the mean frequency, $\nu_X^+(t)$, which evaluates the variation in time of the mean up-crossing rate of the time axis, ii) the central frequency, $\omega_{\mathrm{C},X}(t)$, which scrutinizes the variation of the frequency content of the stochastic process with respect to time and iii) the bandwidth parameter $\delta_X(t)$. The three functions introduced before can be evaluated as a function of NGSMs and have been defined, respectively, as[6]:

$$\nu_{u_r}^+(t)=\frac{1}{2\pi}\sqrt{\frac{\lambda_{2,u_ru_r}(t)}{\lambda_{0,u_ru_r}(t)}}; \quad \omega_{\mathrm{C},u_r}(t)=\frac{\mathrm{Re}\left\{\lambda_{1,u_ru_r}(t)\right\}}{\lambda_{0,u_ru_r}(t)}; \quad \delta_{u_r}(t)=\sqrt{1-\frac{\mathrm{Re}\left\{\lambda_{1,u_ru_r}(t)\right\}^2}{\lambda_{0,u_ru_r}(t)\lambda_{2,u_ru_r}(t)}} \tag{15}$$

where the functions $\lambda_{0,u_ru_r}(t)$ and $\lambda_{2,u_ru_r}(t)$ are real ones while $\lambda_{1,u_ru_r}(t)$ is a complex function. These functions are the so-called *NGSM*s[3,4,6,8].

4 NUMERICAL APPLICATION

4.1 Generation of response spectrum compatible artificial fully non-stationary earthquake accelerograms

The first aim of this section is to obtain fully non-stationary artificial earthquakes that are spectrum compatible and able to reproduce the main features of real recorded time histories; it is important to notice that the comparison with only one result cannot be relevant, so it is necessary to select a set of real recorded earthquakes in order to perform a statistical analysis.

The chosen database is the "PEER: Pacific Earthquake Engineering Research Center: NGA Database"; 30 recorded accelerograms have been selected to perform the statistical analysis. In particular, all of them are time histories of seismic events that happened in the Imperial Valley (California)[10]. Since all the events have a magnitude superior than 5.5 and according to the EC8 instructions[11] a spectrum of type 1 (see Figure 1a) is chosen as target spectrum. The *peak ground acceleration* (*PGA*) is assumed equal to the average of the peak ground acceleration of the recorded events $a_g = 2.483\,\mathrm{m/s^2}$ and, for the type "C" of soil, the parameters $S = 1.15$, $T_B = 0.2\mathrm{s}$, $T_C = 0.6\mathrm{s}$ and $T_D = 2.0\mathrm{s}$ are selected.

An efficient method to generate stationary artificial spectrum compatible accelerograms was established by Cacciola et al[12]; the main problem is that the stationary model of the ground motion acceleration process is unable to catch the characteristics of real earthquakes, such as the amplitude and frequency modulation of the signal; then the energy of the artificial time history is proportional to the duration of the process itself. So, mathematically speaking, stationary samples have infinite energy. A modified method[10], that is able to generate artificial spectrum compatible earthquakes, for fully non-stationary (when both time and frequency content change) random processes, is herein used, by applying the procedure proposed by Cacciola[13]. Thanks' to this iterative method it is possible to obtain the one-sided *PSD* function of the so-called "embedded" stationary counterpart process, $G_0(\omega)$. Furthermore in order to take into account the main features of seismic ground motion, that is the "build-up" and the "die off" segments as well as a decreasing dominant frequency, Spanos and Solomos[14] proposed the following time-frequency modulating function $a(\omega,t)$:

$$a(\omega,t) = \varepsilon(\omega)\, t \exp\left(-\alpha(\omega)t\right)\mathbb{U}(t) \tag{16}$$

where the functions $\varepsilon(\omega)$ and $\alpha(\omega)$ should be defined analyzing the recorded accelerograms. The fully non-stationary random process $F(t)$ is defined by the one-sided *EPSD* function that can be expressed, in the Priestley[2] representation, as the product between the modulating function and the one-sided PSD function of the "embedded" stationary counterpart:

$$G_{FF}(t,\omega) = \left|a(\omega,t)\right|^2 G_0^{\mathrm{STC},j}(\omega), \qquad \omega \geq 0; \quad G_{FF}(t,\omega) = 0, \qquad \omega < 0. \tag{17}$$

where $G_0^{\mathrm{STC},j}(\omega)$ is the spectrum compatible one-sided *PSD* function at the *j*-th iteration.

Following the iterative procedure the spectrum-compatible PSD function $G_{\ddot{U}_\mathrm{g}}^{\mathrm{STC}}(\omega)$ of the stationary counterpart of the fully non-stationary process $\ddot{U}_\mathrm{g}(t)$ is obtained and compared (in logarithmic scale) in Figure 1b with the PSD $G_{\ddot{U}_\mathrm{g}}^{\mathrm{ST}}(\omega)$ under the hypothesis of stationary spectrum-compatible process.

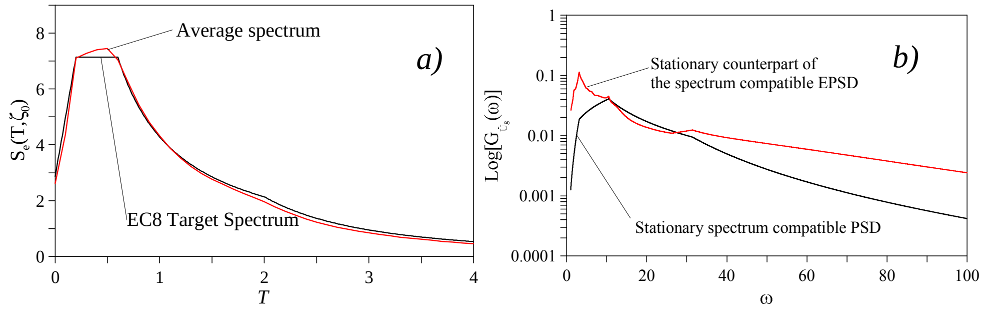

Figure 1: a) Comparison between the selected EC8 target pseudo-acceleration spectrum (black line) and the average of 1000 pseudo-acceleration spectra (red line); b) Spectrum-compatible PSDs. Stationary assumption (black line) and stationary counterpart in the fully non-stationary assumption (red line).

Once the EPSD function of the spectrum-compatible fully non-stationary process is evaluated, a set of 1000 artificial spectrum-compatible fully non-stationary time histories, with duration $t_d = 35\,\mathrm{s}$, are generated by Monte Carlo Simulation. In order to verify the accuracy of the described procedure, in Figure 1a) the EC8[11] target pseudo-acceleration response spectrum with the average pseudo-acceleration response spectrum derived by the 1000 artificial accelerograms are compared.

4.2 Closed form solutions for the *TFR* vector function

After some algebra it can be proved that for the Spanos and Solomos[12] model, whose the time-frequency modulating function, $a(\omega,t)$ is given in Eq.(16), the *TFR vector function*, $\mathbf{Y}_{\mathrm{p}}(\omega,t)$, of the quiescent structural system in modal subspace, forced by the function $f(\omega,t) = \exp(\mathrm{i}\omega t)\,a(\omega,t)$, can be evaluated as:

$$\mathbf{Y}_{\mathrm{p}}(\omega,t) = -\varepsilon(\omega)\exp\left(-\beta(\omega)t\right)\boldsymbol{\Psi}\left[\boldsymbol{\Upsilon}(\omega)+t\,\mathbf{I}_{2m}\right]\boldsymbol{\Upsilon}(\omega)\boldsymbol{\Psi}^T\mathbf{A}\,\mathbf{w}\;\mathbb{U}(t) \tag{18}$$

where $\beta(\omega) = \alpha(\omega) - \mathrm{i}\omega$, and $\boldsymbol{\Upsilon}(\omega)$ is a diagonal matrix defined as $\boldsymbol{\Upsilon}(\omega) = \left[\boldsymbol{\Gamma}+\beta(\omega)\mathbf{I}_{2m}\right]^{-1}$ whose *j*-th element is $\upsilon_j(\omega) = 1/\left[\gamma_j + \beta(\omega)\right]$, being γ_j the *j*-th element of diagonal matrix $\boldsymbol{\Gamma}$. Finally, substituting $\mathbf{Y}_{\mathrm{p}}(\omega,t)$ into Eq.(13) and the result, $\mathbf{Y}(\omega,t)$, into Eq.(8) the explicit closed form solution of the *EPSD* function matrix of the nodal response, in state variable sis obtained.

4.3 Numerical results

In this section in order to verify the accuracy of the proposed procedure the benchmark quiescent linear MDOFs is analysed; this frame has a uniform story height $H = 320$ cm and a bay width $L = 600$ cm, as shown in Figure 2a). The steel columns are made of European HE340A wide flange beams with moment of inertia along the strong axis $I = 27690\ \mathrm{cm}^4$. The steel material is modelled as linear elastic with Young's modulus $E =\ 200$ GPa. The beams are considered rigid to enforce a typical shear building behaviour. Under this assumptions, the shear-frame is modelled as a three DOF linear system. The frame described above is assumed to be part of a building structure with a distance between frames $L_0 = 600$ cm. The tributary mass per story, *M*, is obtained assuming a distributed gravity load of $q = 8\ \mathrm{kN/m^2}$, accounting for the structure's own weight, as well as for permanent and live loads, and it is equal to $M = 28800\,\text{kg}$. The modal periods of the linear elastic undamped shear-frame are $T_1 = 0.376$ s, $T_2 = 0.134$ s and $T_3 = 0.093$ s, with corresponding effective modal mass ratios of 91.41%, 7.45% and 1.10% respectively. The structure presents viscous dampers of coefficient $c = 200$ kNs/m across the second and third stories and a proportional damper of coefficient ηc across the first story. When $\eta = 1$ the structure is classically damped. The elements out of the diagonal can be considered as a measure of the non-classicity of the system. A method to define this value request the introduction of the "coupling index"[15]:

$$\rho = \max\left[\Xi_{i,j}^2 \big/ \left(\Xi_{i,i}\,\Xi_{j,j}\right)\right] \quad (i, j = 1, 2, \ldots, n) \quad i \neq j \quad 0 \leq \rho < 1 \tag{19}$$

As shown in Figure 2b), when $\eta = 1$ the coefficient $\rho = 0$ and the frame is classically damped.

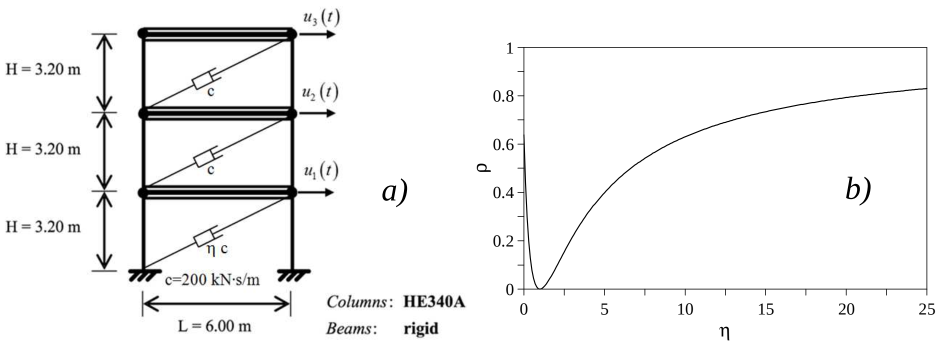

Figure 2: a) Geometric configuration of benchmark three-storey one-bay shear-type frame; b) Coupling index

The benchmark structural model undergoes to a stochastic earthquake base excitation, modelled by a zero mean Gaussian spectrum-compatible fully non-stationary process, as explained in section 4.1. The following figures show the time histories of the mean frequency, the central frequency and the bandwidth parameter of the third floor for $\eta = 1$, $\eta = 10$ and $\eta = 25$ evaluated by the proposed analytical approach and compared with the ones obtained by Monte Carlo Simulation (1000 samples of input).

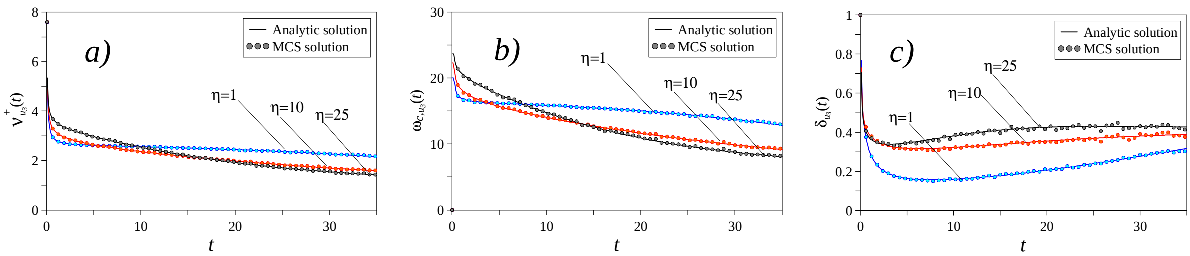

Figure 3: Comparison between the time-variant histories of a) the mean frequency $\nu_{u_3}^{+}(t)$ b) the central frequency $\omega_{C,u_3}(t)$ and c) the bandwidth parameter $\delta_{u_3}(t)$, of the third relative to ground floor displacement, by applying the proposed analytical solution and the Monte Carlo Simulation.

5 CONCLUDING REMARKS

In the framework of stochastic dynamics, in order to perform the safety of structural systems subjected to fully non-stationary input process, the spectral characteristics of the structural response play a fundamental role. These quantities require the evaluation of the NGSMs, which depend on the *TFR* vector function. The main purposes of this paper is to evaluate in explicit closed form the *TFR* vector function of linear non-classically damped structural systems under *fully non-stationary excitation*; the effectiveness of the proposed approach is verified thanks to the comparison with Monte Carlo Simulation results.

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