



## EXPECTED LIFE COST AND OPTIMAL RETROFIT DESIGN OF REINFORCED CONCRETE HOUSING STRUCTURES: A CASE STUDY

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**Key words:** Reinforced concrete, seismic retrofit, vulnerability assessment.

**Parole chiave:** Strutture in calcestruzzo armato, adeguamento sismico, vulnerabilità sismica.

**Abstract.** *A preliminary investigation of a tool for predicting the variation of fragility curves with respect to presumed retrofit costs is presented. Based on Kriging interpolation, the presented approach analyzes the known fragility parameters statistics of observed structural models with respect of their retrofit costs. Considered an objective structure and fixed a retrofit cost, the presented tool estimates the updated fragility parameters as the outcome of a Gaussian process. The proposed strategy proved to be promising as shown by the provided example, although its application in common practice needs further investigations. Benefits and drawbacks are also discussed among with future research developments.*

**Sommario.** *Il presente lavoro introduce uno strumento matematico per la stima di curve di fragilità, relative a strutture esistenti ed ad ipotetici interventi di consolidamento sismico di cui si ipotizza un costo. La procedura si basa sul Kriging ed analizza un opportuno insieme di interventi di consolidamento, noti a priori, di cui vengono preventivamente calcolati il costo di costruzione e le curve di fragilità. Successivamente, data una struttura di cui si conoscono esclusivamente le condizioni pre-intervento, e stabilito un costo di costruzione, l'algoritmo stima, mediante analisi probabilistica, la fragilità post-intervento come occorrenza di un processo Gaussiano. L'approccio, di cui viene presentata una applicazione, si dimostra promettente sebbene necessiti di un maggiore approfondimento prima di poter essere applicato nella pratica professionale. A tal proposito, la sezione conclusiva riporta una sintesi dei principali vantaggi dell'approccio proposto, nonché un'analisi delle problematiche ancora aperte ed una sintesi delle future linee strategiche di ricerca.*

### 1 INTRODUCTION

Seismic retrofit of existing structures become very popular during the last decade in Italy. Catastrophic seismic events occurred recently (e.g., Molise 2002, L'Aquila 2009, Emilia 2012, Amatrice 2016) increased the public awareness about structural safety and the development of efficient and cheap retrofit techniques opened new perspectives about existing structures. Unfortunately, the majority of structural intervention focuses on public and

strategic facilities while private buildings are often forsaken.

The indifference about the safety of private housing structures is a consequence of the fact that rarely a private owner has a precise perspective of retrofit costs as well as a clear understanding of the retrofit benefits both in term of safety and operational costs.

This contribution presents a preliminary study concerning a computational procedure aiming to evaluate the expected variation of the fragility curve of an existing reinforced concrete building, typical of the Italian construction industry of the late '70, subject to different retrofit levels. Specifically, for the considered building, while the funds invested in retrofit increase, different levels of structural safety are reached.

Such fragility esteem can be employed in a subsequent life cost analysis aiming to provide a quantification of the expected damage costs in case of seismic events, computed for each one of the retrofit levels, in order to investigate their economical convenience.

The proposed approach computes structural vulnerability by means of a Monte Carlo based reliability analysis techniques<sup>1,2</sup> where the structure is analyzed by time history analyses, in order to define the probability distribution of structural collapse among 50 years for a fixed location. Therefore, fragility curves are defined by means of a Gaussian distribution whose mean and standard deviation are computed by the results of time history analyses. Parameters of probability distributions are then interpolated by mean of a Kriging interpolation<sup>3</sup> depending on retrofit costs. Provided the initial fragility parameters of an existing building and fixed the retrofit cost, Kriging computes the best unbiased prediction of the expected fragility parameters that the building should present after the retrofit intervention.

Updated fragility curves can be employed in a comparison between pre-event retrofit costs and damage costs in order to identify the optimal retrofit design ensuring a suitable equilibrium between safety and retrofit expensiveness.

The present contribution is organized as follows: retrofit typologies and computation of fragility curves of the *a priori* building set are presented in Section 2 while basics of Kriging interpolation are summarized in Section 3. The specific Kriging formulation employed in this study is therefore presented, along with a numerical example, in Section 4 while Section 5 provides a closure and the conclusions.

## 2 SEISMIC RETROFIT AND FRAGILITY EVALUATION

In order to properly define a set of data for the calibration of the Kriging interpolation, it is necessary to know the fragility curves of a set of building subject to different hypothesis of seismic retrofit intervention. To this end, ten pseudo-experimental reinforced concrete one-directional frame buildings have been considered. Such a specific typology has been chosen since it is typical of Italian construction industry of late '60 and '70 and it proved to be particularly vulnerable to seismic actions in recent earthquake events. A typical configuration of one-directional RC frames, referring to Building 1 of the considered set, is shown in Fig. 1.

For each building, four typologies of retrofit interventions have been designed:

1. Minor retrofit of the existing frames: additional reinforcement bars and cross-section enlargements.
2. Exhaustive retrofit of existing frames: additional reinforcement bars, cross-section enlargement, additional beams linking the one-directional frames.
3. Traditional retrofit: insertion of either steel braces or reinforced concrete shear walls.
4. Innovative retrofit: insertion of either base isolation or dissipative devices.

For each one of the considered structures, retrofit costs, depending on each hypothesized intervention, have been estimated. Moreover, each building has been modeled in the finite-element based framework *OpenSees* in order to analyze both the *a priori* (non-retrofitted) models and the retrofitted ones. Structural models have been modeled by means of force-based frame elements; steel reinforcement constitutive law follows the Menegotto-Pinto model while concrete has been modeled by means of the Kent-Park relationship. Constitutive parameters have been calibrated on usual values employed by construction industry in Italy, namely, class Rck 250 for concrete and class FeB38k for steel reinforcements.

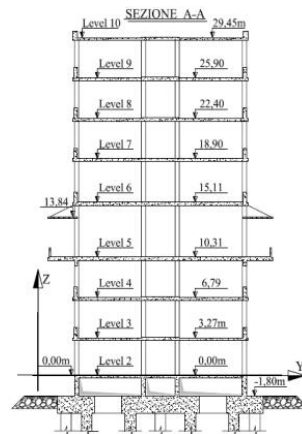


Figure 1: Example of a typical one-directional reinforced concrete frame (Building 1).

Finite element models have been employed in computing fragility curves by means of a Monte Carlo reliability analysis. Specifically, 250 non-stationary ground motions have been artificially generated considering a fixed value of the peak ground acceleration which has been set at  $0.15g$ . Duration of the ground motions has been set at 40 sec. while amplitude non stationarity is taken into account by means of a modulating function. An occurrence of the generated ground motions is shown in Figure 2.

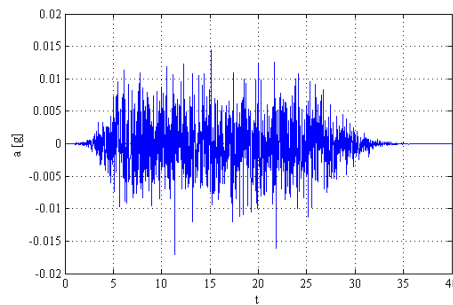


Figure 2: Example of a typical artificially generated ground motion.

The generated ground motions are therefore employed in non-linear time history analyses performed by *OpenSees*. In order to compute the fragility curves of the buildings corresponding to each considered retrofit intervention, a normalized damage variable  $D$ , spanning between 0 (no damage) and 1 (total collapse), has been defined. It depends on the maximum value of top displacements attained during the time history analysis; moreover, the case of total collapse corresponds to the attainment of a lability mechanism of the structural frame.

The whole subset of damage variable, computed for each one of the generated ground

motions, is statistically analyzed in order to get the mean and the variance related to each building and retrofit intervention. Specifically:

$$\begin{aligned} \mu_{j,i} &= E[D_{j,i}] \\ \sigma_{j,i}^2 &= E[(D_{j,i} - E[D_{j,i}])^2] \end{aligned} \quad (1)$$

where  $i=1..10$  denotes the  $i$ -th building;  $j$  denotes either the non-retrofitted model ( $j=0$ ) or the specific considered retrofitting typology ( $j=1..4$ ) as enlisted at the beginning of the present section.

Once that mean and variance have been computed, fragility curves can be approximated by Gaussian distribution. Although different probability distributions provide a better fit of the fragility curves, the Gaussian one is detailed enough for the purposes of the present contribution.

An example of fragility curves related to Building 1 are plotted in Figure 3 where the blue line corresponds to the non-retrofitted building and curves related to the four retrofit typologies have been plotted specifying the unitary retrofit cost  $C$ .

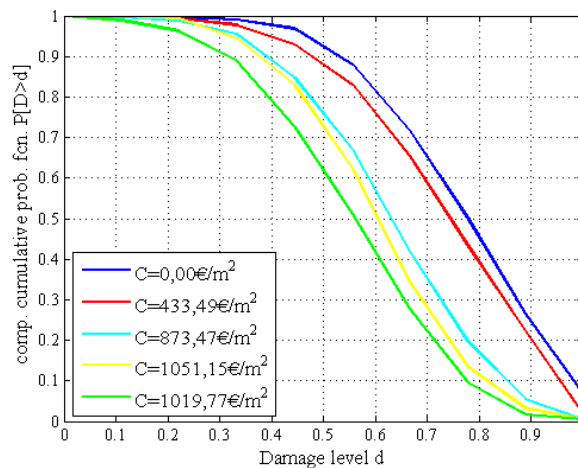


Figure 3: Building 1 Gaussian fragility curves.

Retrofit cost  $C$  has been esteemed for the presumed intervention designed for building 1 and it refers to an unitary area of the building floor. Vulnerability curves are plotted in terms of complementary probability distributions, i.e., fixed a damage level  $d$ , the function plots the probability that the real damage occurred  $D$  is greater than  $d$ . As expected, the blue curve turns out to be plotted at the right of all other curves since the non-retrofitted case corresponds to the higher fragility of the building. As the curves move to the left, the structural fragility decreases so that the green curve, corresponding to a base-isolation intervention, is the safer one. It is worth being emphasized that the greater retrofit cost does not necessarily correspond to a higher structural safety; as a matter of fact, the yellow curve, whose retrofit intervention consists in building a concrete shear wall, turns out to be more expensive to the base-isolation retrofit.

The whole set of fragility curves of the ten “observed” buildings constitutes the *a priori* data of the investigated algorithm and, following an usual practice in Gaussian regression, it is hereafter addressed as “observations”.

### 3 KRIGING INTERPOLATION

Provided the set of fragility curves of the observed buildings, computed in the previous section, this contribution aims to esteem the expected damage mean and variance of an objective structure, fixed its initial, non-retrofitted conditions and the retrofit cost. It is worth being emphasized that the updated fragility curves of the objective structure are not computed by structural analysis; on the contrary, they should be properly derived by fragility curves of the observed structures.

To this end, Kriging<sup>3</sup> is a very effective approach based on Gaussian regression. Originally proposed by Krige, it is widely employed in several contexts involving computer experiments<sup>4</sup> and surrogate finite-element models<sup>5,6</sup>. Its basic idea amounts to predicting the value of a function at a given point as the weighted average of observed data with weights being defined by means of a stochastic model related to the cross-covariance of observations. Main appeal of kriging interpolation consists in its capability to compute unknown function values quite fast regardless of the complexity of the observed data and, at the same time, to provide the estimation of a confidence interval.

The peculiar formulation applied in this contribution aims to numerically predict the value of the mean  $\underline{\mu}$  and variance  $\underline{\sigma}^2$  of the retrofitted fragility curve of the objective structure, fixed the arbitrary retrofit cost  $\underline{C}$ :

$$\begin{aligned}\underline{\mu} &= f_1(\underline{\mu}_0, \underline{\sigma}_0^2, \underline{C}, \mu_{j,i}, \sigma_{j,i}^2) \\ \underline{\sigma}^2 &= f_2(\underline{\mu}_0, \underline{\sigma}_0^2, \underline{C}, \mu_{j,i}, \sigma_{j,i}^2)\end{aligned}\quad (2)$$

where  $\underline{\mu}_0$ ,  $\underline{\sigma}_0^2$  are the mean and variance of the non-retrofitted fragility curve of the objective structure and  $\mu_{j,i}$  and  $\sigma_{j,i}^2$  are means and variances of the observations. To this end, the Kriging predictor is defined, for both the fragility parameters, as:

$$f(\underline{\mu}_0, \underline{\sigma}_0^2, \underline{C}) = m_f + \sum_1^n \lambda_k [f(\mu_{j,i}, \sigma_{j,i}^2) - m_f] \quad (3)$$

where  $f(\underline{\mu}_0, \underline{\sigma}_0^2, \underline{C})$  is the predicted (or *surrogate*) function,  $m_f$  is the function trend,  $f(\mu_{j,i}, \sigma_{j,i}^2)$  are the values of the observations and  $\lambda_k$  are Kriging weights depending on the cross-covariance of the observations. The relationship of Eq. (3) actually consists in a weighted average; thus, Kriging predictor is defined as a weighted regression providing the best, linear unbiased prediction, as expected response of a Gaussian process.

### 4 FRAGILITY CURVES UPDATING PREDICTION

The formulation of the Kriging predictor reported in Eq. (3) is suitably calibrated on the fragility parameters of the observed data. In particular, the trends  $m_{\underline{\mu}}$  and  $m_{\underline{\sigma}}$  of mean  $\underline{\mu}$  and variance  $\underline{\sigma}^2$ , respectively, are assumed to be constant as average of all observations since the responses of the selected buildings are basically uncorrelated. Regression weights  $\lambda_k$  are computed as:

$$\boldsymbol{\lambda} = \mathbf{K}^{-1} \mathbf{k} \quad (4)$$

where  $\boldsymbol{\lambda}$  is a vector of elements  $\lambda_k$ ;  $\mathbf{K}$  is the cross-covariance matrix of the observations and  $\mathbf{k}$  is the cross-covariance at the point  $(\underline{\mu}_0, \underline{\sigma}_0^2, \underline{C})$ . Since statistics of the fragility parameters are not known in closed form, their cross-covariances are estimated by means of Matérn<sup>7</sup> 5/2 models calibrated on the observed data by means of a least-square optimization algorithm.

Computation of the predicted values of fragility parameters is, therefore, straightforward

and consists in an ensemble of linear operations. Kriging predictors of the mean and variance have been calibrated on the fragility parameters of the ten considered buildings. Numerical results are omitted for brevity; nevertheless, Figures 4 and 5 show the surrogate functions of the damage mean and variance-square-root, respectively.

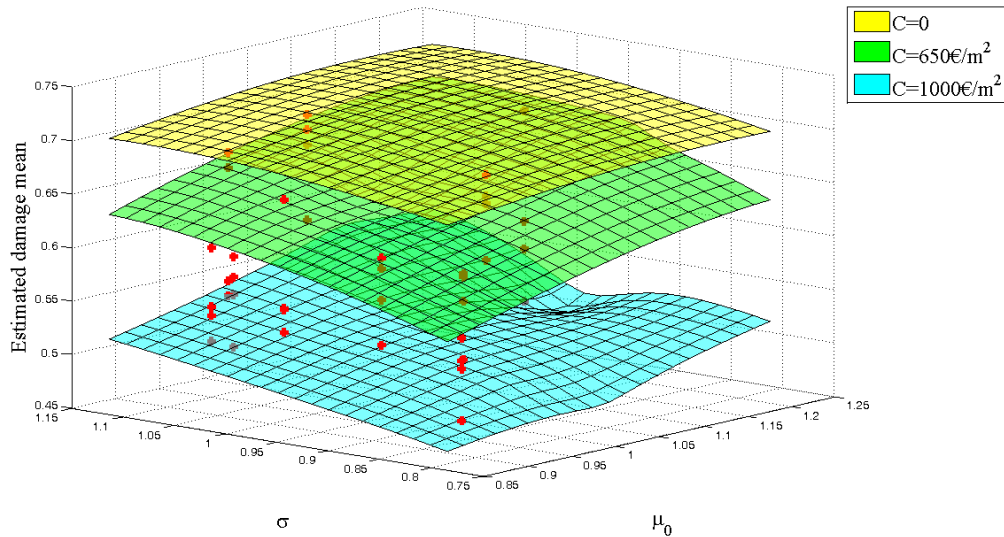


Figure 4: Estimation of the damage mean  $\underline{\mu}$  fixed the retrofit cost.

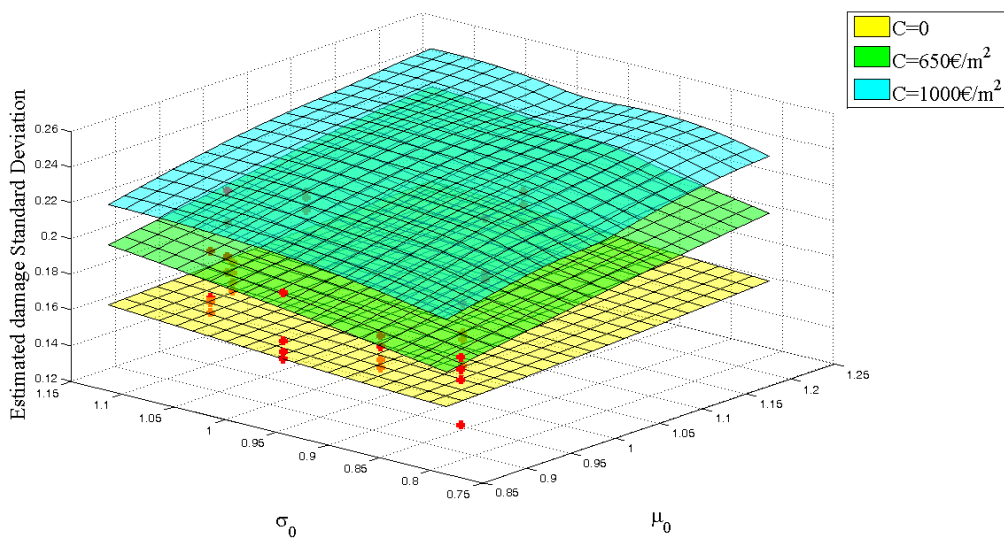


Figure 5: Estimation of the damage standard deviation  $\underline{\sigma}$  fixed the retrofit cost.

Figure 4 shows the prediction of the damage mean  $\underline{\mu}$  depending on the retrofit cost  $\underline{C}$  and on the initial, non-retrofitted conditions  $\underline{\mu}_0$  and  $\underline{\sigma}_0^2$ . Red points represent the mean of the observations' fragility curves while the blue, green and yellow surfaces correspond to three different values of the hypothesized retrofit cost. As expected, the yellow surface, corresponding to  $\underline{C}=0$ , i.e. no retrofit, turns out to provide the higher values of  $\underline{\mu}$ , thus, high damage is more likely to occur. The expected value of the damage tends to decrease as the retrofit cost increases. Analogously, Figure 5 represents the prediction of the damage variance

$\sigma$  depending on retrofit cost and initial conditions. In this case, the higher retrofit cost corresponds to the greater variance of the damage variable. In this sense, the damage probability distribution, as the presumed retrofit cost increases, provides lower and more scattered expected values, so that structural safety is improved, but, at the same time, the actual expected damage has a greater confidence interval. This effect is due to the features of observed data: presumed retrofit interventions used in computing damage statistics are of very different typologies providing very sparse responses. Nevertheless, the prediction of the surrogate fragility parameters allows one to compute the updated fragility curves corresponding to different retrofit costs, as shown in Figure 6.

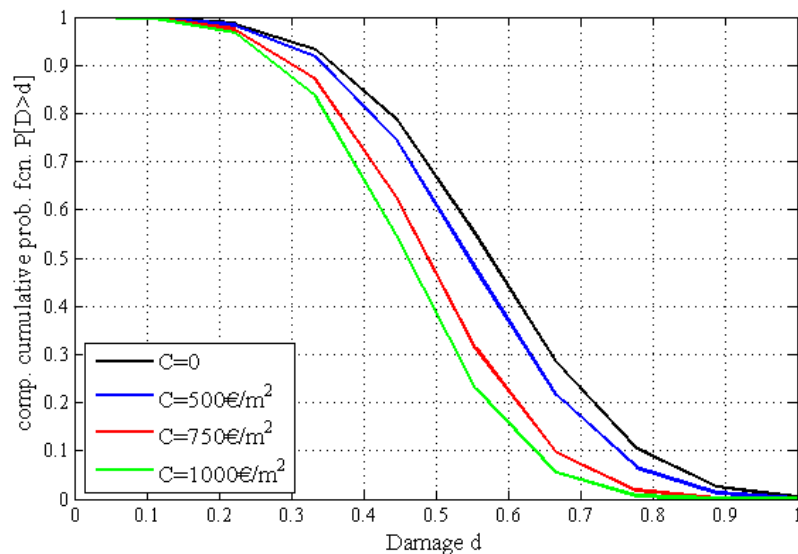


Figure 6: Objective building surrogate fragility curves.

The black line represents the complementary cumulative distribution (CCDF) of the damage variable in case of non-retrofitted conditions. As expected, the predicted CCDF move to lower damage values as the retrofit cost increases.

## 5 CONCLUSIONS AND FUTURE WORK

A preliminary investigation concerning a tool for predicting the variation of fragility curves in case of retrofit intervention has been presented. Updated fragility is computed by means of a Kriging predictor calibrated on a set of observations, i.e., building models for which retrofit interventions have been designed in order to esteem the retrofit cost and to compute the corresponding fragility curves. Although of limited application, because of the small set of observations and the focus on a few fragility parameters, the presented approach proved to address its purpose.

Further extensions are currently under investigations in order to make the tool suitable for a design oriented use in common practice. Specifically, the Kriging predictor will be extended in order to take into account different values of the external seismic action as well as the localization of the considered buildings. Moreover, the knowledge of the objective structure initial fragility can be overcome by introducing typological variables concerning structural features easy to identify, such as construction year, geometrical dimensions and structural typology.

It is worth being emphasized that the Kriging predictor is a very effective tool for dealing with several parameters of different kinds. Thus, the main limitation of the presented formulation concerns the very limited set of observed buildings.

Future research points mainly to arrange an exhaustive database of observations in order to investigate the sensitivity of the Kriging prediction with respect of each considered parameter. Moreover, a larger observation set will provide experimental assessment of the results.

## BIBLIOGRAPHY

- [1] A. Melani, R.K. Khare, R.P. Dhakal, J.B. Mander, “Seismic risk assessment of low rise RC frame structure”, *Structures*, **5**: 13-22, (2016).
- [2] J. Pejovic, S. Jankovic, “Seismic fragility assessment for reinforced concrete high-rise buildings in Southern Euro Mediterranean zone”, *Bulletin of Earthquake Engineering*, **14**: 185-212, (2016).
- [3] D.G. Krige, *A Statistical Approach to Some Mine Valuation and Allied Problems on the Witwatersrand*. M.Sc. Thesis, University of the Witwatersrand, Faculty of Engineering (1951).
- [4] T. Santner, B. Williams, W. Notz, *The Design and Analysis of Computer Experiments*. Springer-Verlag, New York (2003).
- [5] S. Sessa, N. Valoroso, “Use of Kriging to surrogate finite element models of bonded double cantilever beams”. Proc. of ICASP12 – Int. Conf. on Appl. of Stat. and Prob. in Civ. Engrg., Vancouver (2015).
- [6] S. Sessa, N. Valoroso, *Kriging interpolation strategy for Finite-Element-based surrogate responses of DCB delamination tests*, Book chapter in: *Risk and Reliability Analysis: theory and applications – in honor of Prof. Armen Der Kiureghian* ed. P. Gardoni, Springer (2016).
- [7] C. E. Rasmussen, C. K. I. Williams, *Gaussian Processes for Machine Learning*, the MIT Press (2006).