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DICAM

PEDESTRIAN-FOOTBRIDGE DYNAMIC INTERACTION: A PROBABILISTIC ASSESSMENT OF VIBRATION SERVICEABILITY

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Abstract.*Recent footbridges are characterized by long span, light materials and increasing slenderness, which make them more sensitive to dynamic forces induced by pedestrians. The vibration serviceability under human-induced dynamic forces is the governing criterion for the design of these slender structures. To assess serviceability, the bridge response must be computed accounting for human-structure interaction, adopting reliable estimates of expected loading scenarios, accurate models representing pedestrians and suitable integration techniques. The present contribution investigates the dynamic response of a lively footbridge due to a group of pedestrians described as bipedal mechanical systems able to reproduce the human gait. Based on a numerical procedure previously developed, extensive simulation activity is performed in order to characterize the structural response in a probabilistic way. The step frequencies of the pedestrians are considered as correlated random variables, where correlation is based on mutual distance among pedestrians.*

Sommario.*Le passerelle pedonali sono sempre più caratterizzate da luci elevate e materiali leggeri. La combinazione di questi fattori ne aumenta la flessibilità, rendendole più sensibili ai carichi dinamici indotti dai pedoni. La stima delle condizioni in esercizio durante il transito pedonale diventa così il criterio principale per la progettazione di tali opere. Lavalutazione delcomportamento dinamicorichiede che siconsideril'interazione dinamica tra ponte e pedoni assumendo scenari realistici del traffico pedonale, modelli accurati dell'azione del singolo pedone e procedure di integrazione robuste e affidabili.L'articolo analizza la risposta dinamica di una passerella pedonale al passaggio di un gruppo di pedoni, descritti come sistemi meccanici bipedi capaci di riprodurre la locomozione umana. Le simulazioni numeriche, svolte con una procedura precedentemente implementata, mirano a descrivere la risposta del ponte con un approccio probabilistico. Le frequenze del passo di ciascun pedone sono considerate variabili aleatorie correlate tra loro sulla base delle distanze mutue.*

1 INTRODUCTION

New footbridge design is characterized by long spans and light materials resulting in structures with increasing slenderness, prone to pedestrian-induced vibration to be assessed within the class of serviceability limit states[1]. Human-structure interaction(a complex and inter-disciplinary issue)can be subdivided into two key areas[2]. One concerns the influence of structural vibration on forces induced by human occupants;the other the effect of pedestrians on the dynamic properties of civil engineering structures.This paper focuses on the second aspect.

The simplest pedestrian modellingneglects the random variation among human population[3] and assumes that individuals generate identical and perfectly repeatable footfalls. However, the real loading scenario is characterized by pedestrians which never will produce the same footfall, entering the bridge in a random way and able to move undisturbed, each of them with his own characteristics in terms of loading amplitude, frequency, velocity and phase. Thus,a need arises to move from a deterministic to a probabilistic modellingof the pedestrian[4].

A probabilistic approach considersthe individual distinctive gait parameters, whichaffect human-induced forces, such as the body weight, step frequency, etc as random variables having their probability density functions. This approach is able to capture both the inter-variability (different pedestrians generate different forces) and the intra-variability (a single pedestrian induces a different walking force at each step).If a single pedestrian is considered, the walking force can still be assumed periodic and the randomness is introduced through a probabilistic distribution of weight, step frequency, step velocity and so on [5]. On the other hand, for a pedestrian group, the probability distribution of arrival times has to be added. This approach is based on the idea to get a reliable estimate of the force from a group of people by combining forces from individuals[4]. Obviously, for a reliable statistical description of human forces, a large database of measurements is necessary.

In this work, the footbridge-pedestrian system is treated as a coupled dynamic system, where a group of mechanical bipedal models of pedestrians travels along a complete 3D model of the bridge, with the real movement of the human feet. The step frequencies are random variables and their values are derived from a normal distribution. The probabilistic modeladopted in this workgenerates, from a multiple sequences of uncorrelated frequencies, a set of random step frequencies following a relationship described by a given correlation matrix, function of the distances among pedestrians. The bridge response due to the same group of pedestrians is computed for the two cases of correlated and uncorrelated frequencies. A comparison is made in terms of accelerations at selected nodes.

2 BRIDGE DESCRIPTION

The Seriate footbridge in Figure 1a, 63.75 m long, connects two cycle routes in the Serio Park (near to Milan, Italy). The width of the timber deck on a steel grid (Figure 1b) ranges between 2.5 m at the entrance and 5 m at midspan. The slightly curved longitudinal steel girders have a rise of 1.3 m. The transverse beams of the steel grid (Figure 1b) are spaced 1 m apart andare subdivided in main and secondary elements. Main girders have a tapered section and are connected to the hangers. Secondary beams have an IPE 120 cross-section. The stringers are a pair of IPE 330 beams at the edges, and a central beam with a circular section (ϕ=298.5 mm), deemed to stabilize the deck on the horizontal plane.

A series of X-braces (Figure 1b) connects the main transverse girders and provides stiffness in the horizontal plane. The timber deck (Figure 1b) has only a minor structural role, providing the walking surface. The ends of the transverse main girders are crossed by stabilizing cables, whose sliding in the longitudinal direction is allowed by the interposition of a polymeric layer between the two contact surfaces. The suspension system and the backstays cables (connecting the pylons top to the ground) aresupported by the steel main pylons,as shown in Figure 1a. Twomain suspension cables support the longitudinal girders through 42 vertical hangers.Two longitudinalstabilizing cables of opposed curvatures are placed at the sides (Figure 1b).The suspension system is not symmetric neither about the vertical plane crossing the longitudinal bridge axis nor about the vertical plane crossing the midspan. All the cables were pre-tensioned during construction.

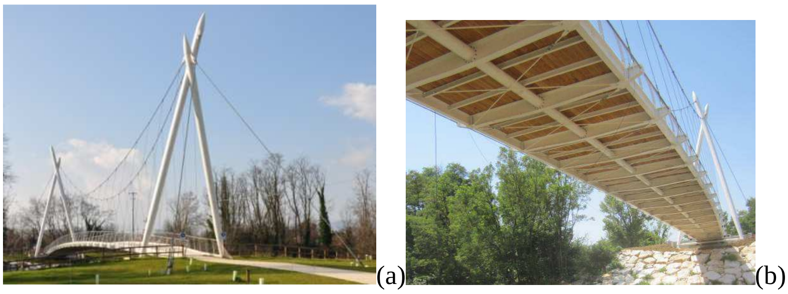

Figure 1. Seriate Footbridge: (a) overall view; (b) footbridge deck, detail of longitudinal and transverse beam.

The dynamic behaviour of the footbridge was investigated through dynamic tests[6]. The bridge response under ambient excitation was recorded with conventional accelerometers. Using output-only techniques typical of the operational modal analysis, 14 vibration modes wereidentified in the frequency range 0-10 Hz. The frequency of the fundamental mode was 1.03 Hz. Figure 2 depicts the mode shapes (identified using the Frequency Domain Decomposition method) that turned out to fall in the frequency range 1.9-3 Hz. Modes 3 to 5 are in the range of walking induced loads, while modes 5 to 7 can be excited by running activities.

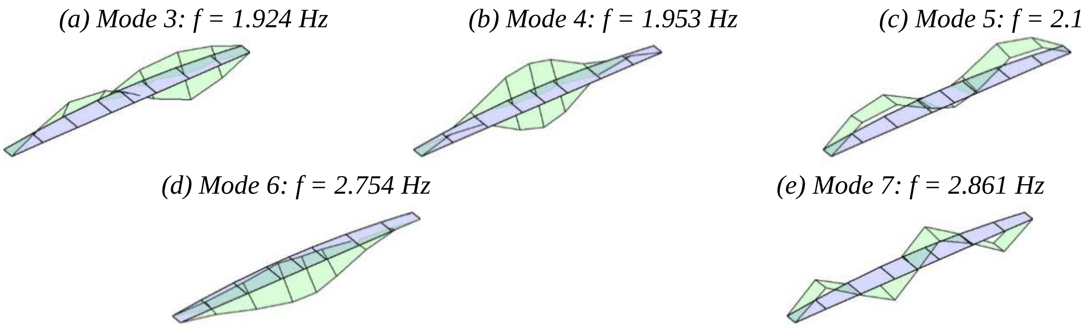

Figure 2. Mode shapes identified: (a) mode 3; (b) mode 4; (c) mode 5; (d) mode 6; (e) mode 7.

3 MECHANICAL MODELFOR THE PEDESTRIAN

A mechanical system is the most accurate pedestrian modelling in the analytical study of human-structure interaction. In this work, reference is made to a bipedal mechanical model recently proposed[7], capable to simulate the human locomotion, characterized by the alternate sequence of Single Support Phase (SSP) and Double Support Phase (DSP) of the typical human gait cycle (Figure 3a). SSP begins when the leading foot hits the ground and the trailing foot is off the ground, moving through the air towards the next position. DSP starts when the trailing foot hits the ground becoming the leading foot and both feet are in contact with the bridge deck.

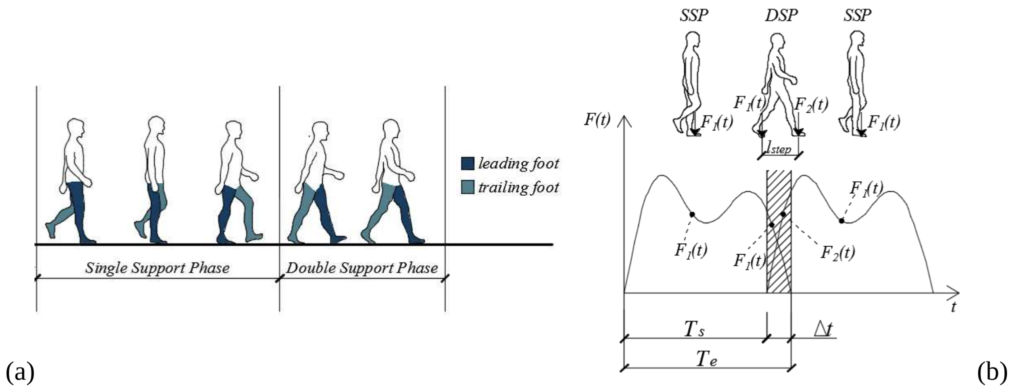

Figure 3. Human gait cycle: (a) sequence of SSP and DSP; (b) single foot force model.

The pedestrian is modelled as a SDOF mass-spring-damper mechanical system. Its equation of motion takes into account the interaction with the bridge and the biomechanical force acting on it. The MSD system has two spring-damper legs (Figure4), capable to reproduce the correct human gait (SSP and DSP in a sequence). The legs parameters follows the data provided by Kim & Park[8]. The mass, lumped at the centre of gravity of the body, oscillates in the vertical direction only. A fictitious bio-mechanical force F_b excites the MSD system. By solving an inverse problem, in this work F_b has been derived as the force that, applied on the system moving on a rigid surface, produces a ground reaction force (transmitted by each leg) matching the single foot force model in Figure 3b.

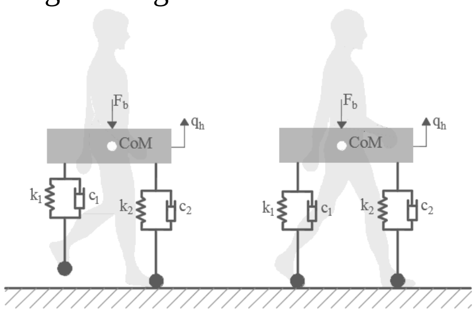

Figure 4. MSD model: left, only one leg is in contact, right, both legs are in contact.

4 EQUATIONS OF MOTION FOR THE COUPLED SYSTEM

The coupled equations of motion governing the HSI are written for the overall system composed of the bridge and the pedestrian mechanical system, where the bridge is discretized with the FE method. Hence, the problem has a finite number of coordinates. The two systems interact at contact points exchanging a vertical contact force: the bridge, due to its flexibility, oscillates and applies a motion at the base of pedestrian’s feet.The walking pedestrian, excited by the biomechanical force and the motion applied by the bridge, transmits to the bridge deck a set of contact forces (leg internal forces). The coupled equations are first forcibly uncoupled and then solved via an iterative procedure[7],[9]. The pedestrian model and the uncoupled algorithm are implemented in a research code named INTER 2.0[7].

4.1 Pedestrian traffic and spatial configurations in INTER 2.0

Inter 2.0 canconsider either a single pedestrian or a group of pedestrian. In both cases the pedestrian’s trajectory is a straight line parallel to the bridge axis, not necessarily coinciding with a line of nodes. The groups of pedestrians can be distributed according to three different spatial configurations: uniform grid (Figure 5a), “chessboard” (Figure 5b) and “random-like” distribution (Figure 5c). Pedestrians move with the same step velocity and their mutual (deterministic) distance do not change during the analysis.

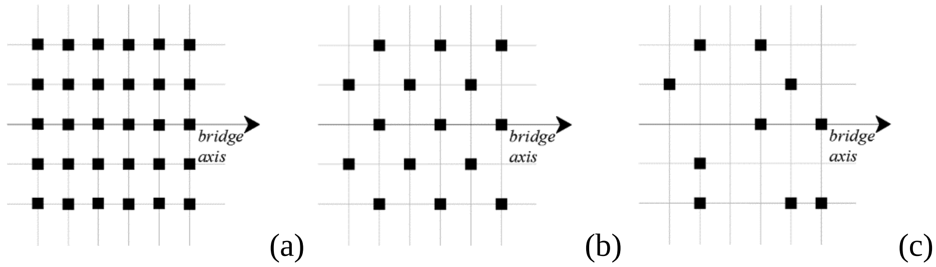

Figure 5 – Pedestrians’ spatial configuration: (a) uniform grid; (b) “chessboard” grid;(c) “random-like” grid.

Three types of synchronization have been considered: full, partial and no synchronization, the only compatible with the probabilistic approach.A total synchronization takes place if *n*identical pedestrians walk perfectly in phase.Partial synchronization means in-phase walking along rows and uncorrelated phases among rows. No synchronization means that a group of *n* pedestrians walks with totally uncorrelated phases. The degree of synchronization is independent of the spatial configuration.

4.2 Step frequencies: probabilistic model

The probabilistic approach is based on the correlation existing between pedestrians’ step frequencies, that are random variables with values derived from a normal distribution. The human’s gait is strongly affected by the presence of other people nearby. Two or more people walking close to each other tend to adapt their velocity, step frequency and obviously step length. On the base of this rationale, the main gait parameters (velocity, frequency and length) can be correlated with the pedestrians’ mutual distances. The correlation decreases as the distance increases. In this work: (a) the step velocity has been assumed constant and equal for all pedestrian; (b) the step length is given by the ratio between step velocity and step frequency; and (c) the step frequency is treated as a random variable, whose correlation structure depends on mutual distances among individuals.

Let’s consider a group of *n* pedestrian whose positions on bridge deck (*x*, *y* coordinates) are known. Their mutual Euclidean distancesare assembled into a symmetrical $n \times n$ matrix $\boldsymbol{d}$, having null diagonal termsd_{ii} and strictly positive out-of-diagonal terms d_{ij}. The correlation coefficient r_{ij} follows an exponential law, where the exponent α is evaluated by imposing a 1% correlation when the distance d_{ij} is equal to the total length L of the bridge deck:

$$r_{ij} = \mathrm{e}^{-\alpha|\mathrm{d_{ij}}|} \tag{1}$$

$$\alpha = -\frac{\ln{(1\%)}}{L} \tag{2}$$

Following equation (1), the correlation matrix r is positive definite and symmetric:

$$\boldsymbol{r} = \begin{bmatrix} 1 & r_{12} & \dots & r_{1n} \\ r_{21} & 1 & & \vdots \\ \vdots & & \ddots & \\ r_{n1} & \dots & & 1 \end{bmatrix} \tag{3}$$

The matrix $\boldsymbol{f}$ of uncorrelated frequencies can be drawn from a normal distribution whose mean value, μ, and standard deviation, σ, have been extracted by literature data[10]. The matrix $\boldsymbol{f}$ has dimensions $k \times n$, where k is the number of samples to be generated and n is the total number of pedestrian.The n sets of correlated frequencies can be obtained by applying the Cholesky decomposition of the correlation matrix into the product of a lower L or upper U triangular matrix by its conjugate transpose[11][12]:

$$\boldsymbol{r} = \boldsymbol{U}\boldsymbol{U}^{\boldsymbol{T}} = \boldsymbol{L}\boldsymbol{L}^{\boldsymbol{T}} \tag{4}$$

Finally, the correlated random matrix of frequencies$\boldsymbol{f_C}$, based on equation (3), can be generated by multiplying the uncorrelated matrix $\boldsymbol{f}$ by $\boldsymbol{U}$:

$$\boldsymbol{f_C} = \boldsymbol{f}\boldsymbol{U} \tag{5}$$

5. RESULTS

Two cases are analyzed, with a uniform distribution of 9 pedestrians (Figure 6a), with weight of 700N, height of 1.70 m and step velocity of 1.30 m/s. Step frequencies are either (partially) correlated or uncorrelated. The number of samples adopted in the probabilistic model is equal to 10 for each case. Pedestrians enter the bridge (Figure 6b) from the left side. The relevant response parameters are the maximum vertical accelerations, evaluated in seven transverse sections of the deck for three nodes, one on the longitudinal axis and two at the bases of handrails(Figure 6b).

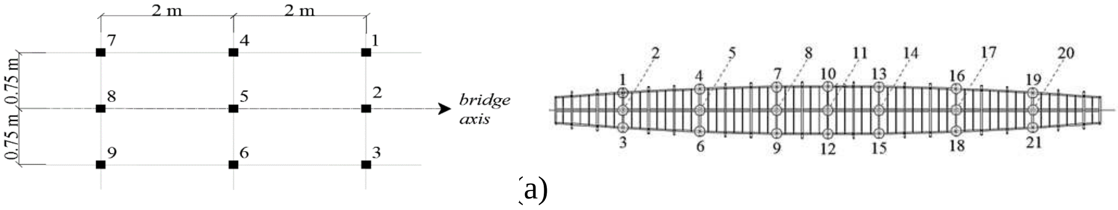

Figure 6 – (a) Pedestrians' spatial configuration; (b)Cross-sections and numbering of nodes.

The comparison between the two models, uncorrelated and correlated, is shown in Table 1 in terms of mean value of maximum accelerations, standard deviation and percentage difference between the mean values of maximum acceleration, computed as it follows:

$$\Delta\mu = (\mu_{correlated} - \mu_{uncorrelated}) * 100/\mu_{correlated} \tag{6}$$

The mean values of maximum accelerationsfor the nodes along the bridge axiswith the two set of frequencies are shown in Figure 7. As a general trend, larger accelerations are found with the uncorrelated approach. This behaviour could be explained with the fact that the uncorrelated frequency matrix is much more scattered, so each sample contains at least a step frequency close to a bridge frequency. In addition, uncorrelated load patterns tend to excite higher modes, which are responsible for the largest contributions to acceleration values.

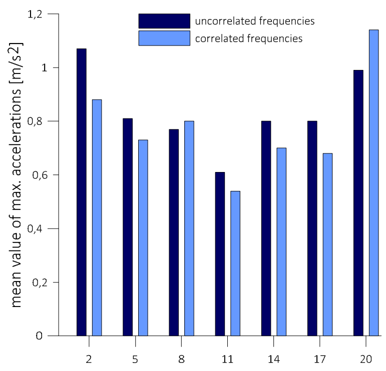

Figure 7 – Mean values of maximum acceleration, bridge axis.

Bridge section	Node	Vertical acceleration [m/s^2]				Δμ [%]
		Uncorrelated frequencies		Correlated frequencies		
		μ	σ	μ	σ	
1	1	1.04	0.13	0.86	0.19	-20.93
	2	1.07	0.12	0.88	0.18	-21.59
	3	1.07	0.12	0.89	0.19	-20.22
2	4	0.79	0.18	0.71	0.34	-11.27
	5	0.81	0.17	0.73	0.33	-10.96
	6	0.79	0.18	0.71	0.34	-11.27
3	7	0.78	0.21	0.80	0.41	2.50
	8	0.77	0.21	0.80	0.45	3.75
	9	0.76	0.20	0.78	0.45	2.56
4	10	0.59	0.06	0.51	0.21	-15.69
	11	0.61	0.04	0.54	0.22	-12.96
	12	0.60	0.05	0.52	0.21	-15.38
5	13	0.79	0.10	0.68	0.29	-16.18
	14	0.80	0.09	0.70	0.28	-14.29
	15	0.82	0.11	0.70	0.27	-17.14
6	16	0.80	0.14	0.66	0.24	-21.21
	17	0.80	0.14	0.68	0.25	-17.65
	18	0.80	0.14	0.66	0.25	-21.21
7	19	1.00	0.25	1.13	0.60	11.50
	20	0.99	0.24	1.14	0.59	13.16
	21	0.98	0.24	1.12	0.60	12.50

Table 1 – Maximum accelerations obtained with the correlated and uncorrelated set of frequencies.

It can be observed that:

- Nodes at section 1 experience larger accelerations adopting the uncorrelated set. The percentage difference is about 20% (mean absolute);
- Nodes at section 2 attain the same results discussed for nodes at section 1, but the percentage difference is lower, about 11% (mean absolute);
- Nodes at section 3 have an opposite trend, the correlated frequencies produce larger accelerations, although the percentage difference is very small, about 3% (mean value);
- Nodes at section 4, 5 and 6 attain the same results discussed for nodes at section 1 and 2, the corresponding percentage differences are about 15%, 16% and 20% (mean absolute), respectively;
- Nodes at section 7 have the same trend of nodes at section 3: the correlated frequencies produce larger accelerations and the percentage difference is about 12% (mean absolute).
- Variance is higher in the cross-sections where the correlated frequencies produce larger accelerations.

12 CONCLUSIONS

In this paper the response of a lively footbridge to walking pedestrians is computed with an uncoupled approach. The bridge is modeled with the Finite Element Method. The pedestrian is represented by a mechanical bipedal model, simulating the sequence of SSP and DSP, typical of the human gait. The two systems interacts at contact points that are not necessarily coincident with bridge mesh nodes. Real walking pedestrians exhibits intra- and inter-variability. In this work, intra-variability is not simulated. The introduction of a probabilistic model for the generation of sequences of correlated random step frequencies takes into account the inter-variability existing among different pedestriansin a partial way. The probabilistic model adopted in this work generates, from multiple sequences of uncorrelated frequencies, a set of random step frequencies following a relationship described by a given correlation matrix, function of the distances among pedestrians.

The numerical analyses concern a group of nine pedestrians placed on a regular grid of three rows and three columns. All of them share a common weight, height and step velocity, while their frequencies are generated from a normal distribution, considering ten samples. Pedestrians are not synchronized, transmitting different forces for the whole time history. Average values of extreme accelerations on the ten samples are computed at each of the nodes at study. Pedestrians with correlated step frequencies induce accelerations that are, as a general trend, lower than those produced by uncorrelated pedestrians.Thus, a probabilistic approach could be necessary to avoid overestimation of the bridge response.

Further studies are necessary to investigate the effect of other significant parameters, as the spatial distribution of pedestrians, their properties in terms of weight and height, and the properties of the mechanical model adopted in the numerical simulations.

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